Volume of Pyramids

Applications and design

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

A pyramid has volume $V = \frac{1}{3}Bh$, where B is base area and h is perpendicular distance from the apex to the base plane. For rectangular bases, $B = lw$; for triangular bases, $B = \frac{1}{2}bt$. Base-triangle height t, pyramid height h, and face slant height are different measurements. To reverse the formula, use $h = 3V/B$ or $B = 3V/h$.

Worked example

A pyramid has rectangular base 6 by 5 and perpendicular height 9. Find volume.

1. The rectangular base area is $B = 6(5) = 30$ square units.
2. The perpendicular height is 9 units. Substitute $V = \frac{1}{3}Bh = \frac{1}{3}(30)(9) = 90$ cubic units.
3. The volume is 90 cubic units. The matching prism holds 270 cubic units, exactly three times this amount; surface slant height is not used.
Answer: 90 cubic units

Practice Problems

Show your thinking.

1. A pyramid-shaped package has square base side 12 cm and interior height 15 cm. Find its capacity in cubic centimeters.

2. A monument is a rectangular pyramid with base 10 by 7 m and perpendicular height 6 m. Find its volume in cubic meters.

3. A pyramid container holds 240 cm^3 and has base area 60 cm^2. Design its perpendicular interior height.

4. A pyramid-shaped sand pile has base 6 by 5 m and height 3 m. Sand costs $\$18$ per cubic meter. Find total cost.

5. A pyramid of base area 48 cm^2 and height 10 cm is filled to 75% of its full volume. Find liquid volume.

6. Two pyramids of base area 36 cm^2 and height 8 cm are joined base-to-base with no overlap. Find combined volume.

7. A square-pyramid mold has side 9 cm and height 8 cm. Each cast uses 54 cm^3 of material. How many full casts fit its volume?

8. Design a square pyramid of volume 100 cm^3 and height 12 cm. Find base side length.

9. A solid is a 6 by 4 by 5 cm rectangular prism topped with a pyramid sharing its 6 by 4 base and having height 3 cm. Find total volume.

10. A pyramid has volume 180 cm^3 and height 9 cm. A design limit requires base area at most 55 cm^2. Does it meet the limit?

Answer Key

1. $720\ \text{cm}^3$
2. $140\ \text{m}^3$
3. 12 cm
4. $\$540$
5. $120\ \text{cm}^3$
6. $192\ \text{cm}^3$
7. 4
8. 5 cm
9. $144\ \text{cm}^3$
10. No; base area is $60\ \text{cm}^2$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The square base area is $B = 12^2 = 144\text{cm}^2$.

2. The perpendicular height is 15 cm. Substitute $V = \frac{1}{3}Bh = \frac{1}{3}(144)(15) = 720\ \text{cm}^3$.

3. The volume is $720\ \text{cm}^3$. The matching prism holds $2160\ \text{cm}^3$, exactly three times this amount; surface slant height is not used.

Problem 2

1. The rectangular base area is $B = 10(7) = 70\text{m}^2$.

2. The perpendicular height is 6 m. Substitute $V = \frac{1}{3}Bh = \frac{1}{3}(70)(6) = 140\ \text{m}^3$.

3. The volume is $140\ \text{m}^3$. The matching prism holds $420\ \text{m}^3$, exactly three times this amount; surface slant height is not used.

Problem 3

1. The capacity requires $240 = \frac{1}{3}(60)h = 20h$.

2. Dividing gives $h = 240/20 = 12$ cm.

3. The check $\frac{1}{3}(60)(12) = 240$ cm^3 verifies the design; interior dimensions determine capacity.

Problem 4

1. The base area is $6(5) = 30$ m^2. The pile volume is $\frac{1}{3}(30)(3) = 30$ m^3.

2. The cost is $30(18) = 540$ dollars.

3. The total is $\$540$. The price multiplies volume, not the base area; these happen to share the numerical value 30 here.

Problem 5

1. The full volume is $\frac{1}{3}(48)(10) = 160$ cm^3.

2. The liquid is $0.75(160) = 120$ cm^3.

3. This percentage is explicitly a fraction of volume, not of height. The remaining empty capacity is $160 - 120 = 40$ cm^3.

Problem 6

1. Each pyramid volume is $\frac{1}{3}(36)(8) = 96$ cm^3.

2. The two nonoverlapping interiors give $96 + 96 = 192$ cm^3.

3. The shared base has volume 0, so the corrected total remains $192 - 0 = 192$ cm^3; no material overlaps at the join.

Problem 7

1. The mold volume is $\frac{1}{3}(9^2)(8) = \frac{1}{3}(81)(8) = 216$ cm^3.

2. The material count is $216/54 = 4$ full casts.

3. Four casts use $4(54) = 216$ cm^3 exactly, so there is no leftover material and no upward rounding.

Problem 8

1. The required base area is $B = 3(100)/12 = 25$ cm^2.

2. The square side satisfies $s^2 = 25$, so the positive side length is $s = 5$ cm.

3. The check $\frac{1}{3}(5^2)(12) = 100$ cm^3 verifies the intended capacity and perpendicular height.

Problem 9

1. The prism holds $6(4)(5) = 120$ cm^3. The shared base area is $6(4) = 24$ cm^2.

2. The pyramid adds $\frac{1}{3}(24)(3) = 24$ cm^3, so total volume is $120 + 24 = 144$.

3. The parts meet only at a face, so their interior volumes do not overlap. The pyramid height is the additional 3 cm above the prism.

Problem 10

1. Recover the base area: $B = 3V/h = 3(180)/9 = 60$ cm^2.

2. The limit comparison is $60 \le 55$, which is false; the excess is $60 - 55 = 5$ cm^2.

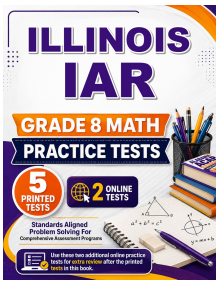
3. The design fails the base-area limit even though its volume and height are consistent with $\frac{1}{3}(60)(9) = 180$.

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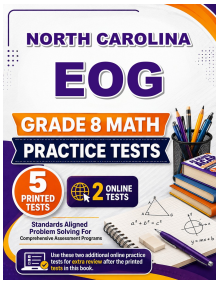
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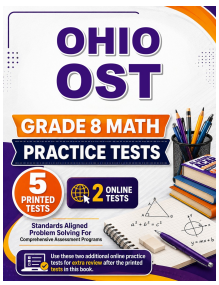
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