Volume of Cylinders, Cones, and Spheres

Applications and design

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Use $V = \pi r^2 h$ for a cylinder, $V = \frac{1}{3}\pi r^2 h$ for a cone, and $V = \frac{4}{3}\pi r^3$ for a sphere. Here r is radius, not diameter, and h is perpendicular height, not slant height. Equal-base, equal-height cones have one third the cylinder volume. Keep π exact until the final approximation; volume uses cubic units.

Worked example

Find the volume of a cone with radius 3 and perpendicular height 7. Give the exact volume in terms of π and an approximation to the nearest tenth.

1. The radius is $r = 3$ units; it is already a radius, so do not halve it.
2. Substitute the dimensions into the cone formula: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(7) = 21\pi$ cubic units.
3. The exact volume is 21π cubic units. Using calculator π gives 66.0 cubic units; round only this final result.
Answer: $21\pi \approx 66.0$ cubic units

Practice Problems

Show your thinking.

1. A cylindrical can has interior radius 3 cm and height 10 cm. Find its capacity in cubic centimeters. Give the exact volume in terms of π and an approximation to the nearest tenth.

2. A conical cup has interior diameter 8 cm and perpendicular depth 9 cm. Find its capacity in cubic centimeters. Give the exact volume in terms of π and an approximation to the nearest tenth.

3. A ball has radius 5 cm. Find its volume in cubic centimeters. Give the exact volume in terms of π and an approximation to the nearest tenth.

4. A cylindrical tank has radius 2 m and height 6 m. It is filled to 75% of its capacity. Find exact water volume.

5. A solid combines a cylinder and a cone joined externally along their full bases of radius 3 cm. The cylinder height is 5 cm and cone perpendicular height is 4 cm. Find exact total volume.

6. A sphere of radius 3 cm is melted into cones of radius 1 cm and height 3 cm. How many full cones can be made without loss?

7. Design a cylindrical container of radius 4 cm to hold exactly 160π cm^3. Find its interior height.

8. A cylindrical pipe has outer radius 3 cm, inner radius 2 cm, and length 10 cm. Find the exact volume of its material.

9. A hemispherical bowl has interior radius 6 cm. Find capacity to the nearest cubic centimeter, using calculator π.

10. A cylinder of radius 2 cm and height 9 cm pours into cones of radius 2 cm and height 3 cm. How many cones can be filled exactly?

Answer Key

1. $90\pi \approx 282.7$ cm^3
2. $48\pi \approx 150.8$ cm^3
3. $\frac{500}{3}\pi \approx 523.6$ cm^3
4. 18π m^3
5. 57π cm^3
6. 36 cones
7. 10 cm
8. 50π cm^3
9. 452 cm^3
10. 9 cones

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The radius is $r = 3$ cm; it is already a radius, so do not halve it.
2. Substitute the dimensions into the cylinder formula: $V = \pi r^2 h = \pi(3)^2(10) = 90\pi$ cm^3.
3. The exact volume is 90π cm^3. Using calculator π gives 282.7 cm^3; round only this final result.

Problem 2

1. The diameter is 8 cm, so $r = 8/2 = 4$ cm.
2. Substitute the dimensions into the cone formula: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2(9) = 48\pi$ cm^3.
3. The exact volume is 48π cm^3. Using calculator π gives 150.8 cm^3; round only this final result.

Problem 3

1. The radius is $r = 5$ cm; it is already a radius, so do not halve it.
2. Substitute the dimensions into the sphere formula: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5)^3 = \frac{500}{3}\pi$ cm^3.
3. The exact volume is $\frac{500}{3}\pi$ cm^3. Using calculator π gives 523.6 cm^3; round only this final result.

Problem 4

1. The full capacity is $\pi(2)^2(6) = 24\pi$ m^3.
2. The filled portion is $0.75(24\pi) = 18\pi$ m^3.
3. The water occupies three quarters of the total 24π, leaving $24\pi - 18\pi = 6\pi$ m^3 empty.

Problem 5

1. The cylinder volume is $\pi(3)^2(5) = 45\pi$ cm^3.
2. The cone adds $\frac{1}{3}\pi(3)^2(4) = 12\pi$ cm^3, giving $45\pi + 12\pi = 57\pi$.
3. The joined solids have 0 overlapping volume, so subtract 0: $57\pi - 0 = 57\pi$ cm^3. Their shared face has no thickness.

Problem 6

1. The sphere contains $\frac{4}{3}\pi(3)^3 = 36\pi$ cm^3.
2. Each cone holds $\frac{1}{3}\pi(1)^2(3) = \pi$ cm^3, so $36\pi/\pi = 36$.
3. Exactly 36 full cones use all the material; no rounding or leftover-volume adjustment is needed because the quotient is an integer.

Problem 7

1. The required capacity gives $160\pi = \pi(4)^2 h = 16\pi h$.
2. Divide by the base area: $h = 160\pi/(16\pi) = 10$ cm.
3. The design check $\pi(4)^2(10) = 160\pi$ cm^3 verifies the exact capacity; use interior dimensions for a container.

Problem 8

1. The outer cylinder volume is $\pi(3)^2(10) = 90\pi$ cm^3.
2. The empty inner cylinder is $\pi(2)^2(10) = 40\pi$ cm^3, so material volume is $90\pi - 40\pi = 50\pi$.
3. Both cylinders have length 10 cm. Subtracting radii before squaring would not measure the annular material correctly.

Problem 9

1. A full sphere of radius 6 has volume $\frac{4}{3}\pi(6)^3 = 288\pi$ cm^3.
2. The hemisphere holds half: $\frac{1}{2}(288\pi) = 144\pi \approx 452.389$ cm^3.
3. Round to 452 cm^3. The flat top closes the bowl opening but contributes no extra volume.

Problem 10

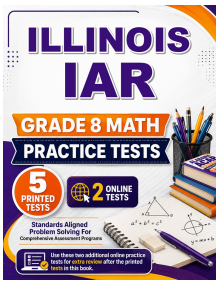
1. The cylinder contains $\pi(2)^2(9) = 36\pi$ cm^3.
2. One cone holds $\frac{1}{3}\pi(2)^2(3) = 4\pi$ cm^3. Divide: $36\pi/(4\pi) = 9$.
3. Nine full cones use exactly the available liquid. The different heights matter as well as the cylinder-to-cone factor of 3.

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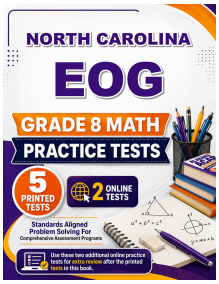
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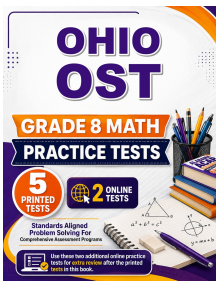
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