Using a Linear Model

Spot mistakes, reason, and work backward

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

To recover an input from a target prediction, solve the model equation instead of treating the target as an input. With $\hat{y} = 2x + 7$ and target 21, $2x = 14$ gives $x = 7$. Residual order matters: actual minus predicted. Check a proposed answer in the original model, and avoid interpreting an intercept outside the data range as an observed fact.

Worked example

Example for form 2: use the prediction rule. Use $\hat{y} = 2x + 6$. Predict the output at $x = 3$.

1. Substitute $x = 3$ into the fitted rule: $\hat{y} = 2(3) + 6$. The product is 6.

2. Combine the intercept to get $6 + 6 = 12$. This is the model prediction at the requested input, not a guaranteed measured result.

Answer: $\hat{y} = 12$

Practice Problems

Show your thinking.

1. Check whether the learner omitted the intercept. Use $\hat{y} = 3x + 7$. A learner predicts 12 at $x = 4$. Check the prediction.

2. Check a prediction with a negative rate. Use $\hat{y} = -2x + 20$. A learner predicts 26 at $x = 3$. Check the prediction.

3. A learner subtracts in reverse order. Find actual minus predicted. Use $\hat{y} = 4x + 2$. At $x = 5$ the actual output is 19. Find actual minus predicted.

4. Recover the input; do not substitute the target as x. Use $\hat{y} = 5x - 3$. Find x when $\hat{y} = 17$.

5. Check a proposed exact prediction. Use $\hat{y} = 2x + 9$. A learner predicts 21 at $x = 6$. Check the prediction.

6. A learner calls the intercept the rate. State both correctly. Use $\hat{y} = 6x + 13$. State the signed rate and initial predicted output.

7. Check the sign of this predicted change. Use $\hat{y} = -3x + 40$. Find the predicted change from $x = 2$ to $x = 7$.

8. A learner says zero residual means zero output. Calculate the residual. Use $\hat{y} = 3x + 1$. At $x = 4$ the actual output is 13. Find actual minus predicted.

9. Recover an input for a falling trend. Use $\hat{y} = -4x + 36$. Find x when $\hat{y} = 16$.

10. For $\hat{y} = 2x + 5$, compare the predictions at $x = 3$ and $x = 6$. Does doubling the input double the prediction?

Answer Key

1. Correct prediction: 19.
2. Correct prediction: 14.
3. -3
4. $x = 4$
5. Correct prediction: 21.
6. Rate 6; initial output 13.
7. -15
8. 0
9. $x = 5$
10. No: predictions 11 and 17, but $2(11) = 22 \neq 17$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Substitute the specified input into the full model: $\widehat{y} = 3(4) + 7 = 19$.

2. The claimed value differs by $12 - (19) = -7$. The nonzero difference shows the claim does not match this model.

Problem 2

1. Substitute the specified input into the full model: $\widehat{y} = -2(3) + 20 = 14$.

2. The claimed value differs by $26 - (14) = 12$. The nonzero difference shows the claim does not match this model.

Problem 3

1. The predicted output at this input is $\widehat{y} = 4(5) + 2 = 22$. Use this, not the intercept alone.

2. The residual is $y - \widehat{y} = 19 - (22) = -3$. The actual result is below the prediction.

Problem 4

1. Set the target equal to the prediction: $17 = 5x - 3$. Subtract the intercept: $20 = 5x$.

2. Divide by 5 to obtain $x = 20/5 = 4$. Checking gives $5(4) - 3 = 17$, the requested model output.

Problem 5

1. Substitute the specified input into the full model: $\widehat{y} = 2(6) + 9 = 21$.

2. The claimed value differs by $21 - (21) = 0$. It agrees with the model.

Problem 6

1. At $x = 0$, $\widehat{y} = 6(0) + 13 = 13$. At $x = 1$, the prediction is 19.

2. The one-unit change is $19 - (13) = 6$. Thus the coefficient 6 is the signed rate and the constant 13 is the starting prediction.

Problem 7

1. The endpoint predictions are $\widehat{y}(2) = -3(2) + 40 = 34$ and $\widehat{y}(7) = -3(7) + 40 = 19$.

2. Subtract final minus initial: $19 - (34) = -15$. Equivalently, rate times input change is $-3(7 - 2) = -15$.

Problem 8

1. The predicted output at this input is $\widehat{y} = 3(4) + 1 = 13$. Use this, not the intercept alone.

2. The residual is $y - \widehat{y} = 13 - (13) = 0$. The actual result equals the prediction.

Problem 9

1. Set the target equal to the prediction: $16 = -4x + 36$. Subtract the intercept: $-20 = -4x$.

2. Divide by -4 to obtain $x = -20/-4 = 5$. Checking gives $-4(5) + 36 = 16$, the requested model output.

Problem 10

1. Evaluate both inputs: $\widehat{y}(3) = 2(3) + 5 = 11$ and $\widehat{y}(6) = 2(6) + 5 = 17$. The second input is twice the first.

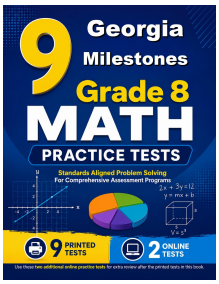
2. Doubling the first prediction gives $2(11) = 22$, not 17. Therefore doubling the input does not double the output; the fixed $+5$ intercept is added only once in each prediction.

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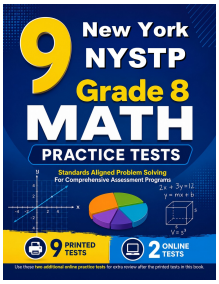
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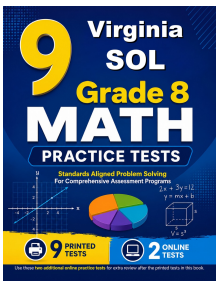
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