Using a Linear Model

Explore ideas and representations

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QUICK REVIEW

A linear model $\widehat{y} = mx + b$ predicts an output by substitution. For $\widehat{y} = 3x + 4$, input 5 predicts 19. The slope describes predicted change per input unit; the intercept is the prediction at input zero. Compare a measured value y with its prediction using residual $y - \widehat{y}$. A fitted prediction is an estimate, not an exact law.

Worked example

Example for form 1: use the prediction rule. Use $\widehat{y} = 2x + 5$. Predict the output at $x = 3$.

1. Substitute $x = 3$ into the fitted rule: $\widehat{y} = 2(3) + 5$. The product is 6.
2. Combine the intercept to get $6 + 5 = 11$. This is the model prediction at the requested input, not a guaranteed measured result.
Answer: $\widehat{y} = 11$

Practice Problems

Show your thinking.

1. Evaluate this fitted relationship. Use $\widehat{y} = 3x + 4$. Predict the output at $x = 5$.

2. Use a decreasing prediction rule. Use $\widehat{y} = -2x + 30$. Predict the output at $x = 6$.

3. Interpret the two model parameters. Use $\widehat{y} = 4x + 7$. State the signed rate and initial predicted output.

4. Calculate the vertical prediction error. Use $\widehat{y} = 2x + 5$. At $x = 4$ the actual output is 16. Find actual minus predicted.

5. Recover an input for a target. Use $\widehat{y} = 3x + 2$. Find x when $\widehat{y} = 20$.

6. Find a predicted output difference. Use $\widehat{y} = 5x + 1$. Find the predicted change from $x = 2$ to $x = 6$.

7. Evaluate a rule with a negative intercept. Use $\widehat{y} = 4x - 6$. Predict the output at $x = 7$.

8. Identify a negative rate and starting value. Use $\widehat{y} = -3x + 25$. State the signed rate and initial predicted output.

9. Find a residual below the line. Use $\widehat{y} = 2x + 8$. At $x = 3$ the actual output is 11. Find actual minus predicted.

10. Solve a decreasing model backward. Use $\widehat{y} = -2x + 24$. Find x when $\widehat{y} = 8$.

Answer Key

1. $\widehat{y} = 19$
2. $\widehat{y} = 18$
3. Rate 4; initial output 7.
4. 3
5. $x = 6$
6. 20
7. $\widehat{y} = 22$
8. Rate -3; initial output 25.
9. -3
10. $x = 8$

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Substitute $x = 5$ into the fitted rule: $\widehat{y} = 3(5) + 4$. The product is 15.

2. Combine the intercept to get $15 + 4 = 19$. This is the model prediction at the requested input, not a guaranteed measured result.

Problem 2

1. Substitute $x = 6$ into the fitted rule: $\widehat{y} = -2(6) + 30$. The product is -12.

2. Combine the intercept to get $-12 + 30 = 18$. This is the model prediction at the requested input, not a guaranteed measured result.

Problem 3

1. At $x = 0$, $\widehat{y} = 4(0) + 7 = 7$. At $x = 1$, the prediction is 11.

2. The one-unit change is $11 - (7) = 4$. Thus the coefficient 4 is the signed rate and the constant 7 is the starting prediction.

Problem 4

1. The predicted output at this input is $\widehat{y} = 2(4) + 5 = 13$. Use this, not the intercept alone.

2. The residual is $y - \widehat{y} = 16 - (13) = 3$. The actual result is above the prediction.

Problem 5

1. Set the target equal to the prediction: $20 = 3x + 2$. Subtract the intercept: $18 = 3x$.

2. Divide by 3 to obtain $x = 18/3 = 6$. Checking gives $3(6) + 2 = 20$, the requested model output.

Problem 6

1. The endpoint predictions are $\widehat{y}(2) = 5(2) + 1 = 11$ and $\widehat{y}(6) = 5(6) + 1 = 31$.

2. Subtract final minus initial: $31 - (11) = 20$. Equivalently, rate times input change is $5(6 - 2) = 20$.

Problem 7

1. Substitute $x = 7$ into the fitted rule: $\widehat{y} = 4(7) - 6$. The product is 28.

2. Combine the intercept to get $28 - 6 = 22$. This is the model prediction at the requested input, not a guaranteed measured result.

Problem 8

1. At $x = 0$, $\widehat{y} = -3(0) + 25 = 25$. At $x = 1$, the prediction is 22.

2. The one-unit change is $22 - (25) = -3$. Thus the coefficient -3 is the signed rate and the constant 25 is the starting prediction.

Problem 9

1. The predicted output at this input is $\widehat{y} = 2(3) + 8 = 14$. Use this, not the intercept alone.

2. The residual is $y - \widehat{y} = 11 - (14) = -3$. The actual result is below the prediction.

Problem 10

1. Set the target equal to the prediction: $8 = -2x + 24$. Subtract the intercept: $-16 = -2x$.

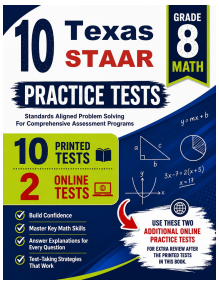
2. Divide by -2 to obtain $x = -16/-2 = 8$. Checking gives $-2(8) + 24 = 8$, the requested model output.

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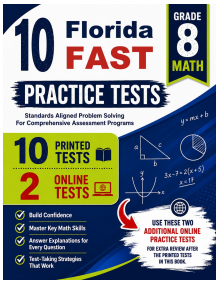
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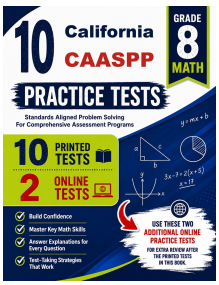
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