Turning Repeating Decimals into Fractions

Exact Repeating Measurements

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

An exact repeating measurement can be stored as a fraction without rounding. For $x = 1.\overline{06}$, the equations $100x = 106.0606\ldots$ and $x = 1.0606\ldots$ give $99x = 105$, hence $x = \frac{35}{33}$. Retain units after converting. Align complete repeating blocks, including zeros, and distinguish a mathematical repeating model from a finite instrument display.

Worked example

A model length is exactly $1.\overline{42}$ m. Express it as a reduced fraction.

1. Let $x = 1.\overline{42}$. Moving past the nonrepeating prefix gives $1x = 1.424242\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 142.424242\ldots$. Subtract the aligned equations: $(100 - 1)x = 142 - 1 = 141$.

3. Thus $99x = 141$ and $x = \frac{141}{99} = \frac{47}{33}$. This reduced fraction is exact; no decimal rounding was used.

Answer: $\frac{47}{33}$ meters.

Practice Problems

Show your thinking.

1. A machine uses exactly $0.\overline{6}$ L per cycle. Find the exact amount used in 9 cycles.

2. Each stripe is exactly $0.4\overline{6}$ m wide. Can 3 stripes fit within 1.5 m? Find their total width.

3. A simulation records an exact probability $0.\overline{125}$. Write the probability as a reduced fraction.

4. A gear advances exactly $0.0\overline{27}$ revolution per tick. How many ticks make 3 complete revolutions?

5. A segment is exactly $1.\overline{23}$ cm long. Design a chain of 3 equal segments and give its exact total length.

6. A designer needs the exact fractional equivalent of $0.7\overline{2}$ m, not the rounded 0.72 m. Find it.

7. A game awards exactly $0.\overline{15}$ point per move. Choose $\frac{5}{33}$ or $\frac{3}{20}$ and verify by subtraction.

8. A pump model adds exactly $0.06\overline{8}$ L each second. Write the modeled rate as a fraction.

9. A fabric cut needs exactly $2.2\overline{7}$ m. Is $\frac{9}{4}$ m enough? Compare exact fractions.

10. A game designer wants an exact chance displayed as $0.\overline{81}$. What reduced fraction should the code store?

Answer Key

1. 6 L.
2. Yes; total 1.4 m.
3. $\frac{125}{999}$
4. 110 ticks.
5. $\frac{122}{33}$ cm.
6. $\frac{13}{18}$ meters.
7. $\frac{5}{33}$
8. $\frac{31}{450}$ liters per second.
9. No; short by $\frac{1}{36}$ m.
10. $\frac{9}{11}$

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Let $x = 0.\overline{6}$. Moving past the nonrepeating prefix gives $1x = 0.666\ldots$; the repeating tail is unchanged.

2. One full block farther gives $10x = 6.666\ldots$. Subtract the aligned equations: $(10 - 1)x = 6 - 0 = 6$.

3. Thus $9x = 6$ and $x = \frac{6}{9} = \frac{2}{3}$. This reduced fraction is exact; no decimal rounding was used.

4. Nine cycles use $9(\frac{2}{3}) = 6$ liters. The fractional rate gives an exact whole-number total without rounding the repeating decimal.

Problem 2

1. Let $x = 0.4\overline{6}$. Moving past the nonrepeating prefix gives $10x = 4.666\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 46.666\ldots$. Subtract the aligned equations: $(100 - 10)x = 46 - 4 = 42$.

3. Thus $90x = 42$ and $x = \frac{42}{90} = \frac{7}{15}$. This reduced fraction is exact; no decimal rounding was used.

4. Three widths total $3(\frac{7}{15}) = \frac{7}{5} = 1.4$ meters. Since $1.4 < 1.5$, they fit, with 0.1 meter left.

Problem 3

1. Let $x = 0.\overline{125}$. Moving past the nonrepeating prefix gives $1x = 0.125125125\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 125.125125125\ldots$. Subtract the aligned equations: $(1000 - 1)x = 125 - 0 = 125$.

3. Thus $999x = 125$ and $x = \frac{125}{999} = \frac{125}{999}$. This reduced fraction is exact; no decimal rounding was used.

Problem 4

1. Let $x = 0.0\overline{27}$. Moving past the nonrepeating prefix gives $10x = 0.272727\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 27.272727\ldots$. Subtract the aligned equations: $(1000 - 10)x = 27 - 0 = 27$.

3. Thus $990x = 27$ and $x = \frac{27}{990} = \frac{3}{110}$. This reduced fraction is exact; no decimal rounding was used.

4. The advance is $\frac{3}{110}$ revolution per tick. The needed count is $3 \div \frac{3}{110} = 110$ ticks, giving exactly 3 revolutions.

Problem 5

1. Let $x = 1.\overline{23}$. Moving past the nonrepeating prefix gives $1x = 1.232323\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 123.232323\ldots$. Subtract the aligned equations: $(100 - 1)x = 123 - 1 = 122$.

3. Thus $99x = 122$ and $x = \frac{122}{99} = \frac{122}{99}$. This reduced fraction is exact; no decimal rounding was used.

4. Three identical segments give $3(\frac{122}{99}) = \frac{122}{33}$ centimeters. The chain length is exact; 1.23 per segment would underestimate it.

Problem 6

1. Let $x = 0.7\overline{2}$. Moving past the nonrepeating prefix gives $10x = 7.222\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 72.222\ldots$. Subtract the aligned equations: $(100 - 10)x = 72 - 7 = 65$.

3. Thus $90x = 65$ and $x = \frac{65}{90} = \frac{13}{18}$. This reduced fraction is exact; no decimal rounding was used.

Problem 7

1. Let $x = 0.\overline{15}$. Moving past the nonrepeating prefix gives $1x = 0.151515\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 15.151515\ldots$. Subtract the aligned equations: $(100 - 1)x = 15 - 0 = 15$.

3. Thus $99x = 15$ and $x = \frac{15}{99} = \frac{5}{33}$. This reduced fraction is exact; no decimal rounding was used.

Problem 8

1. Let $x = 0.06\overline{8}$. Moving past the nonrepeating prefix gives $100x = 6.888\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 68.888\ldots$. Subtract the aligned equations: $(1000 - 100)x = 68 - 6 = 62$.

3. Thus $900x = 62$ and $x = \frac{62}{900} = \frac{31}{450}$. This reduced fraction is exact; no decimal rounding was used.

Problem 9

1. Let $x = 2.2\overline{7}$. Moving past the nonrepeating prefix gives $10x = 22.777\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 227.777\ldots$. Subtract the aligned equations: $(100 - 10)x = 227 - 22 = 205$.

3. Thus $90x = 205$ and $x = \frac{205}{90} = \frac{41}{18}$. This reduced fraction is exact; no decimal rounding was used.

4. The need is $\frac{41}{18} = \frac{82}{36}$ meters; available fabric is $\frac{9}{4} = \frac{81}{36}$. It is short by $\frac{1}{36}$ meter, so not enough.

Problem 10

1. Let $x = 0.\overline{81}$. Moving past the nonrepeating prefix gives $1x = 0.818181\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 81.818181\ldots$. Subtract the aligned equations: $(100 - 1)x = 81 - 0 = 81$.

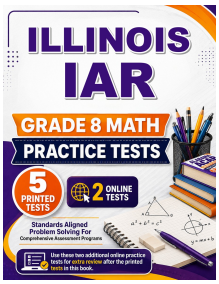
3. Thus $99x = 81$ and $x = \frac{81}{99} = \frac{9}{11}$. This reduced fraction is exact; no decimal rounding was used.

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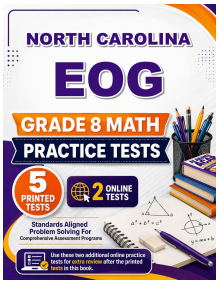
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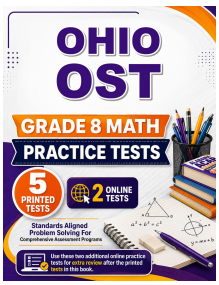
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