Turning Repeating Decimals into Fractions

Repair and Reverse Decimal Conversions

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

The denominator comes from subtraction, not from copying decimal places. If $x = 0.2\overline{3}$, then $100x - 10x = 23 - 2$, so $90x = 21$ and $x = \frac{7}{30}$. A leading zero belongs to its block: $0.\overline{04} = \frac{4}{99}$. Include whole-number parts in both shifted integers, cancel identical tails, and check the reduced result against any proposed fraction.

Worked example

Convert $0.05\overline{4}$, showing why denominator 999 would be wrong.

1. Let $x = 0.05\overline{4}$. Moving past the nonrepeating prefix gives $100x = 5.444\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 54.444\ldots$. Subtract the aligned equations: $(1000 - 100)x = 54 - 5 = 49$.

3. Thus $900x = 49$ and $x = \frac{49}{900} = \frac{49}{900}$. This reduced fraction is exact; no decimal rounding was used.

Answer: $\frac{49}{900}$

Practice Problems

Show your thinking.

1. Correct the claim $0.\overline{7} = \frac{7}{10}$ by using aligned equations.

2. A student writes $0.1\overline{2} = \frac{12}{99}$. Find the correct fraction, accounting for the nonrepeating digit.

3. Why is $0.\overline{09}$ not $0.\overline{9}$? Find the exact fraction for the first number.

4. For $x = 0.04\overline{5}$, choose the useful subtraction: $1000x - 100x$ or $100x - x$. Solve.

5. Find p if $0.\overline{72} = \frac{p}{11}$. First reduce the repeating decimal.

6. For $x = 0.3\overline{18}$, a student gets $990x = 318$. Correct the subtraction and solve.

7. Use subtraction to decide whether $0.\overline{9}$ is less than or equal to 1.

8. Complete and solve: for $x = 0.12\overline{34}$, $10000x - 100x =$ ____.

9. Correct $3.\overline{48} = \frac{348}{99}$. Keep the whole-number part when aligning tails.

10. Find q if $0.8\overline{3} = \frac{5}{q}$. Reduce the exact fraction first.

Answer Key

1. $\frac{7}{9}$
2. $\frac{11}{90}$
3. $\frac{1}{11}$
4. $\frac{41}{900}$
5. $\frac{8}{11}$; hence $p = 8$.
6. $\frac{7}{22}$
7. 1; exactly equal to 1.
8. $\frac{611}{4950}$
9. $\frac{115}{33}$
10. $\frac{5}{6}$; hence $q = 6$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Let $x = 0.\overline{7}$. Moving past the nonrepeating prefix gives $1x = 0.777\ldots$; the repeating tail is unchanged.

2. One full block farther gives $10x = 7.777\ldots$. Subtract the aligned equations: $(10 - 1)x = 7 - 0 = 7$.

3. Thus $9x = 7$ and $x = \frac{7}{9} = \frac{7}{9}$. This reduced fraction is exact; no decimal rounding was used.

Problem 2

1. Let $x = 0.1\overline{2}$. Moving past the nonrepeating prefix gives $10x = 1.222\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 12.222\ldots$. Subtract the aligned equations: $(100 - 10)x = 12 - 1 = 11$.

3. Thus $90x = 11$ and $x = \frac{11}{90} = \frac{11}{90}$. This reduced fraction is exact; no decimal rounding was used.

Problem 3

1. Let $x = 0.\overline{09}$. Moving past the nonrepeating prefix gives $1x = 0.090909\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 9.090909\ldots$. Subtract the aligned equations: $(100 - 1)x = 9 - 0 = 9$.

3. Thus $99x = 9$ and $x = \frac{9}{99} = \frac{1}{11}$. This reduced fraction is exact; no decimal rounding was used.

4. The block has length 2, so $0.\overline{09} = \frac{1}{11}$; using only 9 instead gives $0.\overline{9} = 1$. The leading zero changes the value.

Problem 4

1. Let $x = 0.04\overline{5}$. Moving past the nonrepeating prefix gives $100x = 4.555\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 45.555\ldots$. Subtract the aligned equations: $(1000 - 100)x = 45 - 4 = 41$.

3. Thus $900x = 41$ and $x = \frac{41}{900} = \frac{41}{900}$. This reduced fraction is exact; no decimal rounding was used.

Problem 5

1. Let $x = 0.\overline{72}$. Moving past the nonrepeating prefix gives $1x = 0.727272\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 72.727272\ldots$. Subtract the aligned equations: $(100 - 1)x = 72 - 0 = 72$.

3. Thus $99x = 72$ and $x = \frac{72}{99} = \frac{8}{11}$. This reduced fraction is exact; no decimal rounding was used.

4. Compare the reduced fraction $\frac{8}{11}$ with $\frac{p}{11}$. Equal denominators require $p = 8$, the missing numerator.

Problem 6

1. Let $x = 0.3\overline{18}$. Moving past the nonrepeating prefix gives $10x = 3.181818\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 318.181818\ldots$. Subtract the aligned equations: $(1000 - 10)x = 318 - 3 = 315$.

3. Thus $990x = 315$ and $x = \frac{315}{990} = \frac{7}{22}$. This reduced fraction is exact; no decimal rounding was used.

Problem 7

1. Let $x = 0.\overline{9}$. Moving past the nonrepeating prefix gives $1x = 0.999\ldots$; the repeating tail is unchanged.

2. One full block farther gives $10x = 9.999\ldots$. Subtract the aligned equations: $(10 - 1)x = 9 - 0 = 9$.

3. Thus $9x = 9$ and $x = \frac{9}{9} = 1$. This reduced fraction is exact; no decimal rounding was used.

Problem 8

1. Let $x = 0.12\overline{34}$. Moving past the nonrepeating prefix gives $100x = 12.343434\ldots$; the repeating tail is unchanged.

2. One full block farther gives $10000x = 1234.343434\ldots$. Subtract the aligned equations: $(10000 - 100)x = 1234 - 12 = 1222$.

3. Thus $9900x = 1222$ and $x = \frac{1222}{9900} = \frac{611}{4950}$. This reduced fraction is exact; no decimal rounding was used.

Problem 9

1. Let $x = 3.\overline{48}$. Moving past the nonrepeating prefix gives $1x = 3.484848\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 348.484848\ldots$. Subtract the aligned equations: $(100 - 1)x = 348 - 3 = 345$.

3. Thus $99x = 345$ and $x = \frac{345}{99} = \frac{115}{33}$. This reduced fraction is exact; no decimal rounding was used.

Problem 10

1. Let $x = 0.8\overline{3}$. Moving past the nonrepeating prefix gives $10x = 8.333\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 83.333\ldots$. Subtract the aligned equations: $(100 - 10)x = 83 - 8 = 75$.

3. Thus $90x = 75$ and $x = \frac{75}{90} = \frac{5}{6}$. This reduced fraction is exact; no decimal rounding was used.

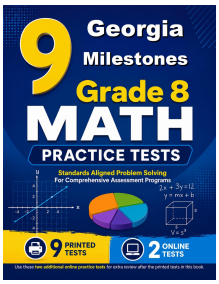
4. The reduced result is $\frac{5}{6}$. Comparing $\frac{5}{q} = \frac{5}{6}$ gives $q = 6$, since the common nonzero numerator is 5.

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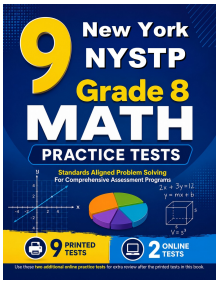
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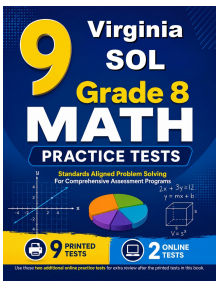
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