Turning Repeating Decimals into Fractions

Align the Repeating Tails

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Let x equal the entire repeating decimal. Multiply by a power of 10 so the repeating tails align, then subtract. For $x = 0.\overline{24}$, $100x - x = 24$, so $99x = 24$ and $x = \frac{8}{33}$. A nonrepeating prefix requires two shifted equations. Reduce the final fraction; keep leading zeros when counting the repeating block length.

Worked example

Convert $0.2\overline{6}$ to a fraction in simplest form.

1. Let $x = 0.2\overline{6}$. Moving past the nonrepeating prefix gives $10x = 2.666\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 26.666\ldots$. Subtract the aligned equations: $(100 - 10)x = 26 - 2 = 24$.

3. Thus $90x = 24$ and $x = \frac{24}{90} = \frac{4}{15}$. This reduced fraction is exact; no decimal rounding was used.

Answer: $\frac{4}{15}$

Practice Problems

Show your thinking.

1. Convert $0.\overline{4}$ to a fraction in simplest form.

2. Convert $0.\overline{18}$ to a fraction in simplest form.

3. Convert $0.\overline{063}$ to a fraction in simplest form.

4. Convert $0.1\overline{6}$ to a fraction in simplest form.

5. Convert $0.2\overline{45}$ to a fraction in simplest form.

6. Convert $0.03\overline{7}$ to a fraction in simplest form.

7. Convert $2.\overline{36}$ to a fraction in simplest form.

8. Convert $1.4\overline{2}$ to a fraction in simplest form.

9. Which fraction equals $0.\overline{54}$: $\frac{54}{100}$ or $\frac{6}{11}$? Show the conversion.

10. Convert $0.5\overline{0}$ exactly. Explain why this repeating notation also represents a terminating decimal.

Answer Key

1. $\frac{4}{9}$
2. $\frac{2}{11}$
3. $\frac{7}{111}$
4. $\frac{1}{6}$
5. $\frac{27}{110}$
6. $\frac{17}{450}$
7. $\frac{26}{11}$
8. $\frac{64}{45}$
9. $\frac{6}{11}$
10. $\frac{1}{2}$; $0.5000\ldots = 0.5$ terminates.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Let $x = 0.\overline{4}$. Moving past the nonrepeating prefix gives $1x = 0.444\ldots$; the repeating tail is unchanged.

2. One full block farther gives $10x = 4.444\ldots$. Subtract the aligned equations: $(10 - 1)x = 4 - 0 = 4$.

3. Thus $9x = 4$ and $x = \frac{4}{9} = \frac{4}{9}$. This reduced fraction is exact; no decimal rounding was used.

Problem 2

1. Let $x = 0.\overline{18}$. Moving past the nonrepeating prefix gives $1x = 0.181818\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 18.181818\ldots$. Subtract the aligned equations: $(100 - 1)x = 18 - 0 = 18$.

3. Thus $99x = 18$ and $x = \frac{18}{99} = \frac{2}{11}$. This reduced fraction is exact; no decimal rounding was used.

Problem 3

1. Let $x = 0.\overline{063}$. Moving past the nonrepeating prefix gives $1x = 0.063063063\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 63.063063063\ldots$. Subtract the aligned equations: $(1000 - 1)x = 63 - 0 = 63$.

3. Thus $999x = 63$ and $x = \frac{63}{999} = \frac{7}{111}$. This reduced fraction is exact; no decimal rounding was used.

Problem 4

1. Let $x = 0.1\overline{6}$. Moving past the nonrepeating prefix gives $10x = 1.666\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 16.666\ldots$. Subtract the aligned equations: $(100 - 10)x = 16 - 1 = 15$.

3. Thus $90x = 15$ and $x = \frac{15}{90} = \frac{1}{6}$. This reduced fraction is exact; no decimal rounding was used.

Problem 5

1. Let $x = 0.2\overline{45}$. Moving past the nonrepeating prefix gives $10x = 2.454545\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 245.454545\ldots$. Subtract the aligned equations: $(1000 - 10)x = 245 - 2 = 243$.

3. Thus $990x = 243$ and $x = \frac{243}{990} = \frac{27}{110}$. This reduced fraction is exact; no decimal rounding was used.

Problem 6

1. Let $x = 0.03\overline{7}$. Moving past the nonrepeating prefix gives $100x = 3.777\ldots$; the repeating tail is unchanged.

2. One full block farther gives $1000x = 37.777\ldots$. Subtract the aligned equations: $(1000 - 100)x = 37 - 3 = 34$.

3. Thus $900x = 34$ and $x = \frac{34}{900} = \frac{17}{450}$. This reduced fraction is exact; no decimal rounding was used.

Problem 7

1. Let $x = 2.\overline{36}$. Moving past the nonrepeating prefix gives $1x = 2.363636\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 236.363636\ldots$. Subtract the aligned equations: $(100 - 1)x = 236 - 2 = 234$.

3. Thus $99x = 234$ and $x = \frac{234}{99} = \frac{26}{11}$. This reduced fraction is exact; no decimal rounding was used.

Problem 8

1. Let $x = 1.4\overline{2}$. Moving past the nonrepeating prefix gives $10x = 14.222\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 142.222\ldots$. Subtract the aligned equations: $(100 - 10)x = 142 - 14 = 128$.

3. Thus $90x = 128$ and $x = \frac{128}{90} = \frac{64}{45}$. This reduced fraction is exact; no decimal rounding was used.

Problem 9

1. Let $x = 0.\overline{54}$. Moving past the nonrepeating prefix gives $1x = 0.545454\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 54.545454\ldots$. Subtract the aligned equations: $(100 - 1)x = 54 - 0 = 54$.

3. Thus $99x = 54$ and $x = \frac{54}{99} = \frac{6}{11}$. This reduced fraction is exact; no decimal rounding was used.

Problem 10

1. Let $x = 0.5\overline{0}$. Moving past the nonrepeating prefix gives $10x = 5.000\ldots$; the repeating tail is unchanged.

2. One full block farther gives $100x = 50.000\ldots$. Subtract the aligned equations: $(100 - 10)x = 50 - 5 = 45$.

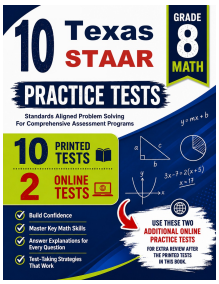
3. Thus $90x = 45$ and $x = \frac{45}{90} = \frac{1}{2}$. This reduced fraction is exact; no decimal rounding was used.

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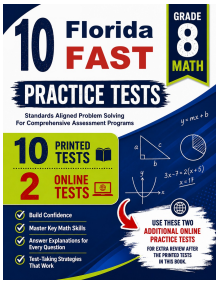
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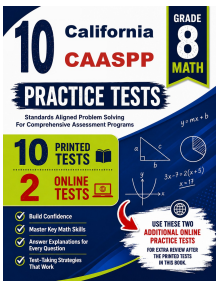
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