Solving Systems of Two Equations

Systems: Build, Compare, and Constrain

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Systems describe two simultaneous constraints, such as a total count and a total value. Define each variable before writing equations and check both conditions afterward. A solution may be mathematically unique yet invalid for whole-item counts. Compare parallel models using elimination: $x + y = 5$ and $2x + 2y = 11$ cannot both hold, while replacing 11 with 10 leaves infinitely many possibilities.

Worked example

There are 8 tickets: adult tickets cost $\$5$ and child tickets $\$3$. Revenue is $\$32$. Find adult count x and child count y. $1x + 1y = 8$, $5x + 3y = 32$.

1. Eliminate x: multiply the first equation by 5 and the second by 1. The resulting x coefficients both equal 5.

2. The scaled equations are $5x + 5y = 40$ and $5x + 3y = 32$. Subtracting the second from the first gives $2y = 8$.

3. Divide to obtain $y = 4$. Substitute into the first equation: $1x + (1)(4) = 8$, giving $x = 4$.

4. Check: $(1)(4) + (1)(4) = 8$; $(5)(4) + (3)(4) = 32$. Both constraints hold, so the pair is unique.

Answer: $(x, y) = (4, 4)$.

Practice Problems

Show your thinking.

1. A box has 10 red and blue counters. Red counters score 2 points and blue 5; total score is 32. Find counts x red and y blue. $1x + 1y = 10$, $2x + 5y = 32$.

2. Two numbers x, y have sum 18 and difference 4. Find them. $1x + 1y = 18$, $1x - 1y = 4$.

3. Bundle A uses one notebook x and one pen y, costing $\$7$. Bundle B uses two notebooks and one pen, costing $\$12$. Find item prices. $1x + 1y = 7$, $2x + 1y = 12$.

4. A design requires $x + y = 9$ and twice the same total to be 20. Can any dimensions satisfy both constraints? $1x + 1y = 9$, $2x + 2y = 20$.

5. A designer requires $x + 2y = 8$ and $3x + 6y = 24$. Do these conditions determine one design? $1x + 2y = 8$, $3x + 6y = 24$.

6. Design two bundle prices for notebooks $n = \$4$ and pens $p = \$1$: use $n + p = A$ and $2n + p = B$. Find A, B and show recovery.

7. A club orders 5 small and large kits. Small kits cost $\$4$ and large $\$6$; the budget is exactly $\$25$. Solve for counts x, y and assess feasibility. $1x + 1y = 5$, $4x + 6y = 25$.

8. Two plans have equal outputs when $y = 2x + 4$ and $y = 5x - 2$. Find the common input and output. $2x - 1y = -4$, $5x - 1y = 2$.

9. A rectangular card has length x and width y, with half-perimeter 11 centimeters and length 3 centimeters greater than width. Find both. $1x + 1y = 11$, $1x - 1y = 3$.

10. One invoice says $2x + y = 10$; a second says $4x + 2y = 23$ for the same item prices. Explain whether the invoices can both be correct. $2x + 1y = 10$, $4x + 2y = 23$.

Answer Key

1. $(x, y) = (6, 4)$.
2. $(x, y) = (11, 7)$.
3. $(x, y) = (5, 2)$.
4. No solution.
5. Infinitely many solutions.
6. $A = \$5$, $B = \$9$; unique prices $(4, 1)$.
7. $(x, y) = (\frac{5}{2}, \frac{5}{2})$. No feasible contextual solution.
8. $(x, y) = (2, 8)$.
9. $(x, y) = (7, 4)$.
10. No solution.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Eliminate x: multiply the first equation by 2 and the second by 1. The resulting x coefficients both equal 2.

2. The scaled equations are $2x + 2y = 20$ and $2x + 5y = 32$. Subtracting the second from the first gives $-3y = -12$.

3. Divide to obtain $y = 4$. Substitute into the first equation: $1x + (1)(4) = 10$, giving $x = 6$.

4. Check: $(1)(6) + (1)(4) = 10$; $(2)(6) + (5)(4) = 32$. Both constraints hold, so the pair is unique.

Problem 2

1. Eliminate x: multiply the first equation by 1 and the second by 1. The resulting x coefficients both equal 1.

2. The scaled equations are $1x + 1y = 18$ and $1x - 1y = 4$. Subtracting the second from the first gives $2y = 14$.

3. Divide to obtain $y = 7$. Substitute into the first equation: $1x + (1)(7) = 18$, giving $x = 11$.

4. Check: $(1)(11) + (1)(7) = 18$; $(1)(11) + (-1)(7) = 4$. Both constraints hold, so the pair is unique.

Problem 3

1. Eliminate x: multiply the first equation by 2 and the second by 1. The resulting x coefficients both equal 2.

2. The scaled equations are $2x + 2y = 14$ and $2x + 1y = 12$. Subtracting the second from the first gives $1y = 2$.

3. Divide to obtain $y = 2$. Substitute into the first equation: $1x + (1)(2) = 7$, giving $x = 5$.

4. Check: $(1)(5) + (1)(2) = 7$; $(2)(5) + (1)(2) = 12$. Both constraints hold, so the pair is unique.

Problem 4

1. Eliminate x: multiply the first equation by 2 and the second by 1. The resulting x coefficients both equal 2.

2. The scaled equations are $2x + 2y = 18$ and $2x + 2y = 20$. Subtracting the second from the first gives $0y = -2$.

3. The result $0 = -2$ is false. No ordered pair can satisfy both equations, so the system has no solution.

Problem 5

1. Eliminate x: multiply the first equation by 3 and the second by 1. The resulting x coefficients both equal 3.

2. The scaled equations are $3x + 6y = 24$ and $3x + 6y = 24$. Subtracting the second from the first gives $0y = 0$.

3. The result $0 = 0$ is true. Both equations describe the same line; all pairs satisfying $1x + 2y = 8$ solve the system.

Problem 6

1. Substitute the chosen prices: $A = 4 + 1 = 5$ and $B = 2(4) + 1 = 9$. The designed system is $n + p = 5$, $2n + p = 9$.

2. Subtract the first equation from the second: $n = 9 - 5 = 4$. Substitute into $n + p = 5$ to get $4 + p = 5$, so $p = 1$.

3. Recovered prices $(4, 1)$ give totals $4 + 1 = 5$ and $2(4) + 1 = 9$. The independent equations determine a unique pair rather than a redundant condition.

Problem 7

1. The count equation is $x + y = 5$ and the value equation is $4x + 6y = 25$. Whole kits require nonnegative integer solutions, not just any real intersection.

2. The scaled equations are $4x + 4y = 20$ and $4x + 6y = 25$. Subtracting the second from the first gives $-2y = -5$.

3. Divide to obtain $y = \frac{5}{2}$. Substitute into the first equation: $1x + (1)(\frac{5}{2}) = 5$, giving $x = \frac{5}{2}$.

4. Check: $(1)(\frac{5}{2}) + (1)(\frac{5}{2}) = 5$; $(4)(\frac{5}{2}) + (6)(\frac{5}{2}) = 25$. Fractional counts are impossible for whole items.

Problem 8

1. Eliminate x: multiply the first equation by 5 and the second by 2. The resulting x coefficients both equal 10.

2. The scaled equations are $10x - 5y = -20$ and $10x - 2y = 4$. Subtracting the second from the first gives $-3y = -24$.

3. Divide to obtain $y = 8$. Substitute into the first equation: $2x + (-1)(8) = -4$, giving $x = 2$.

4. Check: $(2)(2) + (-1)(8) = -4$; $(5)(2) + (-1)(8) = 2$. Both constraints hold, so the pair is unique.

Problem 9

1. Eliminate x: multiply the first equation by 1 and the second by 1. The resulting x coefficients both equal 1.

2. The scaled equations are $1x + 1y = 11$ and $1x - 1y = 3$. Subtracting the second from the first gives $2y = 8$.

3. Divide to obtain $y = 4$. Substitute into the first equation: $1x + (1)(4) = 11$, giving $x = 7$.

4. Check: $(1)(7) + (1)(4) = 11$; $(1)(7) + (-1)(4) = 3$. Both constraints hold, so the pair is unique.

Problem 10

1. Eliminate x: multiply the first equation by 4 and the second by 2. The resulting x coefficients both equal 8.

2. The scaled equations are $8x + 4y = 40$ and $8x + 4y = 46$. Subtracting the second from the first gives $0y = -6$.

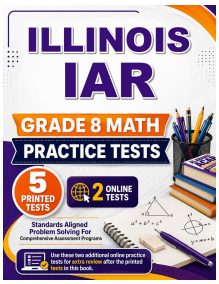
3. The result $0 = -6$ is false. No ordered pair can satisfy both equations, so the system has no solution.

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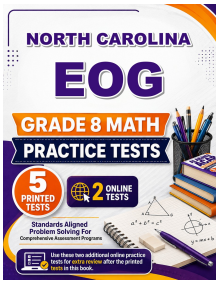
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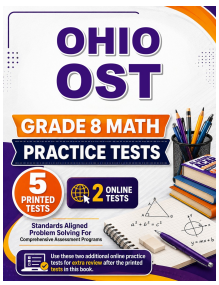
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