Solving Systems of Two Equations

Systems: Diagnose Claims and Design Cases

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Check a claimed solution in both equations, not just one. Equal variable-coefficient ratios can produce either no solutions or infinitely many; compare constants too. If doubling $x + y = 4$ gives $2x + 2y = 8$, the pair is redundant, whereas $2x + 2y = 9$ contradicts it. To design a unique system, choose two nonparallel equations meeting at the intended ordered pair.

Worked example

A student claims $(3, 2)$ solves both equations. Check and correct. $1x + 1y = 5$, $2x + 1y = 9$.

1. The claimed pair satisfies $3 + 2 = 5$, but $2(3) + 2 = 8 \neq 9$. Satisfying only one equation is insufficient, so the claimed solution must be corrected.

2. The scaled equations are $2x + 2y = 10$ and $2x + 1y = 9$. Subtracting the second from the first gives $1y = 1$.

3. Divide to obtain $y = 1$. Substitute into the first equation: $1x + (1)(1) = 5$, giving $x = 4$.

4. Check: $(1)(4) + (1)(1) = 5$; $(2)(4) + (1)(1) = 9$. Both constraints hold, so the pair is unique.

Answer: $(x, y) = (4, 1)$.

Practice Problems

Show your thinking.

1. A student claims $(1, 4)$ solves this system. Diagnose and find the solution. $1x + 1y = 5$, $3x - 1y = 3$.

2. A student concludes $y = 0$ from cancellation here. Explain the correct result. $1x + 2y = 5$, $2x + 4y = 10$.

3. A student calls these equations identical because the left sides scale. Correct the claim. $2x - 1y = 4$, $6x - 3y = 15$.

4. Find k so $x + y = 6$ and $2x + 2y = k$ have infinitely many solutions.

5. Choose k so $x - y = 3$ and $2x - 2y = k$ have no solution. Explain.

6. Design two equations with unique solution $(2, -1)$. Use $x + y$ in one and $x - y$ in the other.

7. Does adding these equations eliminate a variable? Solve and explain. $2x + 3y = 13$, $1x - 3y = -1$.

8. A proposed solution is $(0, 3)$. Verify it or find the correct pair. $1x + 1y = 3$, $2x - 1y = -3$.

9. Explain why different-looking equations need not determine a unique pair. $3x + 1y = 7$, $9x + 3y = 21$.

10. Classify the system before solving, using its coefficient and constant ratios. $1x - 2y = 1$, $3x - 6y = 8$.

Answer Key

1. $(x, y) = (2, 3)$.
2. Infinitely many solutions.
3. No solution.
4. $k = 12$.
5. One choice is $k = 7$.
6. $x + y = 1$ and $x - y = 3$.
7. $(x, y) = (4, \frac{5}{3})$.
8. $(x, y) = (0, 3)$.
9. Infinitely many solutions.
10. No solution.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The first check gives $1 + 4 = 5$, but the second gives $3(1) - 4 = -1 \neq 3$. The pair is not a simultaneous solution.

2. The scaled equations are $3x + 3y = 15$ and $3x - 1y = 3$. Subtracting the second from the first gives $4y = 12$.

3. Divide to obtain $y = 3$. Substitute into the first equation: $1x + (1)(3) = 5$, giving $x = 2$.

4. Check: $(1)(2) + (1)(3) = 5$; $(3)(2) + (-1)(3) = 3$. Both constraints hold, so the pair is unique.

Problem 2

1. Doubling the first equation gives $2x + 4y = 10$, exactly the second. Subtracting identical equations produces $0 = 0$, not the condition $y = 0$.

2. The scaled equations are $2x + 4y = 10$ and $2x + 4y = 10$. Subtracting the second from the first gives $0y = 0$.

3. The result $0 = 0$ is true. Both equations describe the same line; all pairs satisfying $1x + 2y = 5$ solve the system.

Problem 3

1. Triple the first equation: $6x - 3y = 12$. Its left side matches the second, but the second constant is 15, not 12.

2. The scaled equations are $12x - 6y = 24$ and $12x - 6y = 30$. Subtracting the second from the first gives $0y = -6$.

3. The result $0 = -6$ is false. No ordered pair can satisfy both equations, so the system has no solution.

Problem 4

1. Double the entire first equation: $2(x + y) = 2(6)$, giving $2x + 2y = 12$. An identical second equation must have constant 12.

2. Set $k = 12$. Subtracting the doubled first equation from the second leaves $0 = 0$, true for all points of $x + y = 6$.

3. For example, $(0, 6)$ and $(2, 4)$ satisfy both equations. Since the whole line works, the system has infinitely many solutions.

Problem 5

1. Doubling $x - y = 3$ gives $2x - 2y = 6$. To make the second equation inconsistent, its constant must differ from 6.

2. Choose $k = 7$. Subtracting the doubled first equation from $2x - 2y = 7$ gives $0 = 1$, a false numerical statement.

3. No pair can make one expression equal both 6 and 7. Thus the chosen system has no solution; any $k \neq 6$ would work.

Problem 6

1. At the target pair, $x + y = 2 + (-1) = 1$ and $x - y = 2 - (-1) = 3$. Use these numerical values as the two constants.

2. Add the designed equations $x + y = 1$ and $x - y = 3$: $2x = 4$, so $x = 2$. Substitute back: $2 + y = 1$, giving $y = -1$.

3. The two different slopes ensure one intersection. The explicit elimination gives exactly $(2, -1)$, verifying the required unique solution.

Problem 7

1. Addition cancels $3y + (-3y) = 0$, giving $3x = 12$ and $x = 4$. In the first equation, $8 + 3y = 13$, so $y = \frac{5}{3}$.

2. The scaled equations are $2x + 3y = 13$ and $2x - 6y = -2$. Subtracting the second from the first gives $9y = 15$.

3. Divide to obtain $y = \frac{5}{3}$. Substitute into the first equation: $2x + (3)(\frac{5}{3}) = 13$, giving $x = 4$.

4. Check: $(2)(4) + (3)(\frac{5}{3}) = 13$; $(1)(4) + (-3)(\frac{5}{3}) = -1$. Both constraints hold, so the pair is unique.

Problem 8

1. At $(0, 3)$ the first side is $0 + 3 = 3$ and the second is $2(0) - 3 = -3$. Both checks succeed; elimination can confirm uniqueness.

2. The scaled equations are $2x + 2y = 6$ and $2x - 1y = -3$. Subtracting the second from the first gives $3y = 9$.

3. Divide to obtain $y = 3$. Substitute into the first equation: $1x + (1)(3) = 3$, giving $x = 0$.

4. Check: $(1)(0) + (1)(3) = 3$; $(2)(0) + (-1)(3) = -3$. Both constraints hold, so the pair is unique.

Problem 9

1. Multiply the first equation by 3: $9x + 3y = 21$. This is exactly the second equation, so its different appearance adds no independent condition.

2. The scaled equations are $27x + 9y = 63$ and $27x + 9y = 63$. Subtracting the second from the first gives $0y = 0$.

3. The result $0 = 0$ is true. Both equations describe the same line; all pairs satisfying $3x + 1y = 7$ solve the system.

Problem 10

1. The second variable coefficients are 3 times the first ones. But $8 \neq 3(1) = 3$, suggesting parallel distinct lines rather than a common intersection.

2. The scaled equations are $3x - 6y = 3$ and $3x - 6y = 8$. Subtracting the second from the first gives $0y = -5$.

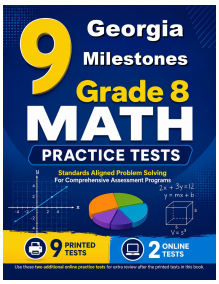
3. The result $0 = -5$ is false. No ordered pair can satisfy both equations, so the system has no solution.

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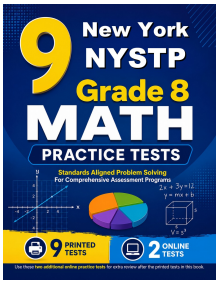
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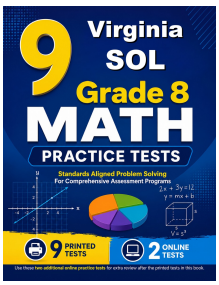
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