Solving Real Problems with Systems

Real Systems: Design and Evaluate Constraints

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Use systems to design an outcome or decide whether a plan is possible. Compute totals from intended values, then solve the resulting equations to recover them. In comparisons, define both a common input and a common output. Check context restrictions after solving: a negative dimension or fractional ticket count is not feasible, even when both algebraic equations are satisfied exactly.

Worked example

Design total revenue for 8 tickets, with 3 adults at $\$7$ and the rest children at $\$4$. Show the resulting system recovers the counts.

1. There are $c = 8 - 3 = 5$ children. Revenue is $7(3) + 4(5) = 21 + 20 = 41$, so use equations $a + c = 8$ and $7a + 4c = 41$.

2. Subtract four times the count equation: $7a + 4c - (4a + 4c) = 41 - 32$, so $3a = 9$ and $a = 3$.

3. Then $c = 8 - 3 = 5$. The recovered whole counts match the design, and checking $21 + 20 = 41$ confirms the revenue.

Answer: $\$41$; $a + c = 8$, $7a + 4c = 41$ gives $(3, 5)$.

Practice Problems

Show your thinking.

1. A field has length x and width y meters, perimeter 64 meters, and length 8 meters more than width. Find dimensions. $1x + 1y = 32$, $1x - 1y = 8$.

2. A snack order has 18 bags. Nuts cost $\$3$, fruit $\$2$, and the exact budget is $\$43$. Find nuts x and fruit y. $1x + 1y = 18$, $3x + 2y = 43$.

3. A 10-kilogram mix uses $\$4$/kilogram grain x and $\$7$/kilogram grain y, totaling $\$52$. Find the two masses in kilograms. $1x + 1y = 10$, $4x + 7y = 52$.

4. Two service plans have cost $y = 5x + 12$ and $y = 8x + 3$ dollars after x uses. Find the break-even input and cost. $5x - 1y = -12$, $8x - 1y = -3$.

5. A workshop claims $2x + 3y = 18$ for materials x, y but also $4x + 6y = 40$. Can any nonnegative amounts satisfy both? $2x + 3y = 18$, $4x + 6y = 40$.

6. A designer specifies $x + 2y = 14$ and $3x + 6y = 42$ for nonnegative real dimensions. Does this determine one design? $1x + 2y = 14$, $3x + 6y = 42$.

7. Design prices for a notebook $x = \$3$ and folder $y = \$2$ by reporting costs of bundles $(2, 1)$ and $(1, 3)$. Show recovery.

8. A boat travels 18 kilometers downstream in 2 hours and 10 kilometers upstream in 2 hours. Let x be still-water speed and y current speed, in kilometers per hour. $1x + 1y = 9$, $1x - 1y = 5$.

9. A school orders 9 whole kits. Standard kits cost $\$4$, premium $\$6$, and exact spending must be $\$43$. Find counts x, y and assess feasibility. $1x + 1y = 9$, $4x + 6y = 43$.

10. A 12-item order uses $\$5$ and $\$2$ items. Design a total cost giving exactly 4 expensive items. Recover both counts.

Answer Key

1. $(x, y) = (20, 12)$.
2. $(x, y) = (7, 11)$.
3. $(x, y) = (6, 4)$.
4. $(x, y) = (3, 27)$.
5. No solution.
6. Infinitely many feasible solutions: $(x, y) = (14 - 2u, u)$, $0 \le u \le 7$.
7. Bundle totals $\$8$ and $\$9$; unique prices $(3, 2)$.
8. $(x, y) = (7, 2)$.
9. $(x, y) = (\frac{11}{2}, \frac{7}{2})$. No feasible contextual solution.
10. $\$36$; expensive 4, inexpensive 8.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Eliminate x: multiply the first equation by 1 and the second by 1. The resulting x coefficients both equal 1.

2. The scaled equations are $1x + 1y = 32$ and $1x - 1y = 8$. Subtracting the second from the first gives $2y = 24$.

3. Divide to obtain $y = 12$. Substitute into the first equation: $1x + (1)(12) = 32$, giving $x = 20$.

4. Check: $(1)(20) + (1)(12) = 32$; $(1)(20) + (-1)(12) = 8$. Both constraints hold, so the pair is unique.

Problem 2

1. Eliminate x: multiply the first equation by 3 and the second by 1. The resulting x coefficients both equal 3.

2. The scaled equations are $3x + 3y = 54$ and $3x + 2y = 43$. Subtracting the second from the first gives $1y = 11$.

3. Divide to obtain $y = 11$. Substitute into the first equation: $1x + (1)(11) = 18$, giving $x = 7$.

4. Check: $(1)(7) + (1)(11) = 18$; $(3)(7) + (2)(11) = 43$. Both constraints hold, so the pair is unique.

Problem 3

1. Eliminate x: multiply the first equation by 4 and the second by 1. The resulting x coefficients both equal 4.

2. The scaled equations are $4x + 4y = 40$ and $4x + 7y = 52$. Subtracting the second from the first gives $-3y = -12$.

3. Divide to obtain $y = 4$. Substitute into the first equation: $1x + (1)(4) = 10$, giving $x = 6$.

4. Check: $(1)(6) + (1)(4) = 10$; $(4)(6) + (7)(4) = 52$. Both constraints hold, so the pair is unique.

Problem 4

1. Eliminate x: multiply the first equation by 8 and the second by 5. The resulting x coefficients both equal 40.

2. The scaled equations are $40x - 8y = -96$ and $40x - 5y = -15$. Subtracting the second from the first gives $-3y = -81$.

3. Divide to obtain $y = 27$. Substitute into the first equation: $5x + (-1)(27) = -12$, giving $x = 3$.

4. Check: $(5)(3) + (-1)(27) = -12$; $(8)(3) + (-1)(27) = -3$. Both constraints hold, so the pair is unique.

Problem 5

1. Eliminate x: multiply the first equation by 4 and the second by 2. The resulting x coefficients both equal 8.

2. The scaled equations are $8x + 12y = 72$ and $8x + 12y = 80$. Subtracting the second from the first gives $0y = -8$.

3. The result $0 = -8$ is false. No ordered pair can satisfy both equations, so the system has no solution.

Problem 6

1. Eliminate x: multiply the first equation by 3 and the second by 1. The resulting x coefficients both equal 3.

2. The scaled equations are $3x + 6y = 42$ and $3x + 6y = 42$. Subtracting the second from the first gives $0y = 0$.

3. With $y = u$, $x = 14 - 2u$. Nonnegativity requires $u \ge 0$ and $14 - 2u \ge 0$, hence $0 \le u \le 7$. This interval gives infinitely many feasible pairs.

Problem 7

1. Compute the bundle totals: $2(3) + 2 = 8$ and $3 + 3(2) = 9$. The price equations are $2x + y = 8$ and $x + 3y = 9$.

2. Double the second equation and subtract the first: $(2x + 6y) - (2x + y) = 18 - 8$, giving $5y = 10$ and $y = 2$.

3. Then $2x + 2 = 8$ gives $x = 3$. Both bundle costs check, so the designed reports uniquely recover the intended item prices.

Problem 8

1. Eliminate x: multiply the first equation by 1 and the second by 1. The resulting x coefficients both equal 1.

2. The scaled equations are $1x + 1y = 9$ and $1x - 1y = 5$. Subtracting the second from the first gives $2y = 4$.

3. Divide to obtain $y = 2$. Substitute into the first equation: $1x + (1)(2) = 9$, giving $x = 7$.

4. Check: $(1)(7) + (1)(2) = 9$; $(1)(7) + (-1)(2) = 5$. Both constraints hold, so the pair is unique.

Problem 9

1. Eliminate x: multiply the first equation by 4 and the second by 1. The resulting x coefficients both equal 4.

2. The scaled equations are $4x + 4y = 36$ and $4x + 6y = 43$. Subtracting the second from the first gives $-2y = -7$.

3. Divide to obtain $y = \frac{7}{2}$. Substitute into the first equation: $1x + (1)(\frac{7}{2}) = 9$, giving $x = \frac{11}{2}$.

4. Check: $(1)(\frac{11}{2}) + (1)(\frac{7}{2}) = 9$; $(4)(\frac{11}{2}) + (6)(\frac{7}{2}) = 43$. Fractional counts are impossible for whole items.

Problem 10

1. With $x = 4$ expensive items, $y = 12 - 4 = 8$ inexpensive items. The designed total is $5(4) + 2(8) = 20 + 16 = 36$ dollars.

2. The equations are $x + y = 12$ and $5x + 2y = 36$. Subtract twice the first equation from the second: $3x = 36 - 24 = 12$.

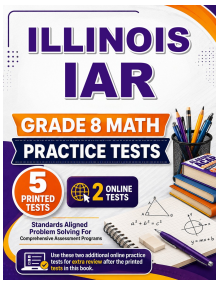
3. Thus $x = 4$ and $y = 12 - 4 = 8$. Both are whole nonnegative counts, and $5(4) + 2(8) = 36$ checks the chosen spending target.

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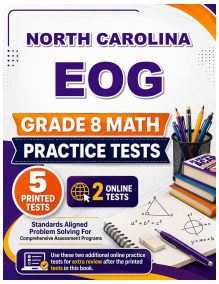
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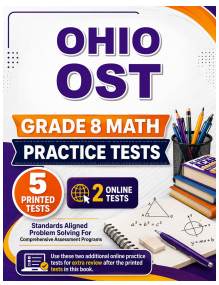
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