Solving Real Problems with Systems

Real Systems: Audit Models and Missing Information

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

An equation must match the variable definitions and units, not just the numbers in a story. For x adult tickets at $\$6$ and y child tickets at $\$4$, revenue is $6x + 4y$, not $4x + 6y$. Verify both totals before accepting a claim. Redundant price equations leave infinitely many real choices; inconsistent invoices have no shared price solution.

Worked example

10 tickets bring $\$44$; adult tickets cost $\$5$, children $\$3$. A student claims $x = 4$ adults, $y = 6$ children. Check and correct. $1x + 1y = 10$, $5x + 3y = 44$.

1. The claimed counts total $4 + 6 = 10$, but revenue is $5(4) + 3(6) = 38 \neq 44$. Matching the count alone does not verify the proposed ticket mix.
2. The scaled equations are $5x + 5y = 50$ and $5x + 3y = 44$. Subtracting the second from the first gives $2y = 6$.
3. Divide to obtain $y = 3$. Substitute into the first equation: $1x + (1)(3) = 10$, giving $x = 7$.
4. Check: $(1)(7) + (1)(3) = 10$; $(5)(7) + (3)(3) = 44$. Both constraints hold, so the pair is unique.
Answer: $(x, y) = (7, 3)$.

Practice Problems

Show your thinking.

1. A stall sells 8 large and small drinks for $\$22$. Large drinks cost $\$4$, small $\$2$. Let x count large drinks. Repair the claim $2x + 4y = 22$ and solve. $1x + 1y = 8$, $4x + 2y = 22$.

2. A rectangle has perimeter 30 and length x exceeding width y by 3. A student uses $x + y = 30$. Correct and solve. $1x + 1y = 15$, $1x - 1y = 3$.

3. Two identical price reports give $x + 3y = 12$ and $2x + 6y = 24$ for nonnegative real prices. Can they determine both prices? $1x + 3y = 12$, $2x + 6y = 24$.

4. A report states one kit x plus two refills y costs $\$11$; three such bundles cost $\$36$. Identify the inconsistency. $1x + 2y = 11$, $3x + 6y = 36$.

5. A box holds 7 pencils and markers priced at $\$1$ and $\$3$ each, totaling $\$12$. Find counts x, y and explain whether the purchase is possible. $1x + 1y = 7$, $1x + 3y = 12$.

6. A ticket report gives $x + y = 9$, with adult price $\$6$ and child price $\$2$. Design revenue so exactly 3 adults attend, and recover the counts.

7. A student says plans $C = 2h + 10$ and $C = 4h + 2$ cost the same at $h = 3$. Test the claim and find equality, using $x = h, y = C$. $2x - 1y = -10$, $4x - 1y = -2$.

8. Two prices satisfy $x + y = 8$. A second report says $2x + 2y = 16$. Add a nonredundant condition giving $x = 5, y = 3$ and solve.

9. A jar has 11 coins worth 85 cents, only nickels and dimes. A claim says 5 nickels and 6 dimes. Check it and determine counts x, y. $1x + 1y = 11$, $5x + 10y = 85$.

10. A positive length x and width y satisfy $x + y = 10$ and $x - y = 14$. Solve and judge whether a rectangle is possible. $1x + 1y = 10$, $1x - 1y = 14$.

Answer Key

1. $(x, y) = (3, 5)$.
2. $(x, y) = (9, 6)$.
3. Infinitely many feasible solutions: $(x, y) = (12 - 3u, u)$, $0 \le u \le 4$.
4. No solution.
5. $(x, y) = (\frac{9}{2}, \frac{5}{2})$. No feasible contextual solution.
6. Revenue \$30; $(x, y) = (3, 6)$.
7. $(x, y) = (4, 18)$.
8. Add $x - y = 2$; unique pair $(5, 3)$.
9. $(x, y) = (5, 6)$.
10. $(x, y) = (12, -2)$. No feasible contextual solution.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Large drinks are counted by x, so their cost is $4x$, not $2x$. The corrected revenue equation is $4x + 2y = 22$, with $x + y = 8$.

2. The scaled equations are $4x + 4y = 32$ and $4x + 2y = 22$. Subtracting the second from the first gives $2y = 10$.

3. Divide to obtain $y = 5$. Substitute into the first equation: $1x + (1)(5) = 8$, giving $x = 3$.

4. Check: $(1)(3) + (1)(5) = 8$; $(4)(3) + (2)(5) = 22$. Both constraints hold, so the pair is unique.

Problem 2

1. Perimeter counts both lengths and both widths: $2x + 2y = 30$. Dividing by 2 gives the correct equation $x + y = 15$, together with $x - y = 3$.

2. The scaled equations are $1x + 1y = 15$ and $1x - 1y = 3$. Subtracting the second from the first gives $2y = 12$.

3. Divide to obtain $y = 6$. Substitute into the first equation: $1x + (1)(6) = 15$, giving $x = 9$.

4. Check: $(1)(9) + (1)(6) = 15$; $(1)(9) + (-1)(6) = 3$. Both constraints hold, so the pair is unique.

Problem 3

1. Eliminate x: multiply the first equation by 2 and the second by 1. The resulting x coefficients both equal 2.

2. The scaled equations are $2x + 6y = 24$ and $2x + 6y = 24$. Subtracting the second from the first gives $0y = 0$.

3. With $y = u$, $x = 12 - 3u$. Nonnegativity requires $u \ge 0$ and $12 - 3u \ge 0$, hence $0 \le u \le 4$. This interval gives infinitely many feasible pairs.

Problem 4

1. Eliminate x: multiply the first equation by 3 and the second by 1. The resulting x coefficients both equal 3.

2. The scaled equations are $3x + 6y = 33$ and $3x + 6y = 36$. Subtracting the second from the first gives $0y = -3$.

3. The result $0 = -3$ is false. No ordered pair can satisfy both equations, so the system has no solution.

Problem 5

1. Eliminate x: multiply the first equation by 1 and the second by 1. The resulting x coefficients both equal 1.

2. The scaled equations are $1x + 1y = 7$ and $1x + 3y = 12$. Subtracting the second from the first gives $-2y = -5$.

3. Divide to obtain $y = \frac{5}{2}$. Substitute into the first equation: $1x + (1)(\frac{5}{2}) = 7$, giving $x = \frac{9}{2}$.

4. Check: $(1)(\frac{9}{2}) + (1)(\frac{5}{2}) = 7$; $(1)(\frac{9}{2}) + (3)(\frac{5}{2}) = 12$. Fractional counts are impossible for whole items.

Problem 6

1. With $x = 3$ adults and 9 tickets total, children number $y = 9 - 3 = 6$. The required revenue is $6(3) + 2(6) = 30$ dollars.

2. The designed system is $x + y = 9$, $6x + 2y = 30$. Subtract twice the first from the second: $4x = 12$, so $x = 3$.

3. Then $y = 9 - 3 = 6$. Both counts are whole and nonnegative, and the check $18 + 12 = 30$ confirms the intended revenue.

Problem 7

1. At $h = 3$, the costs are $2(3) + 10 = 16$ and $4(3) + 2 = 14$. They are unequal, so a different common input is needed.

2. The scaled equations are $8x - 4y = -40$ and $8x - 2y = -4$. Subtracting the second from the first gives $-2y = -36$.

3. Divide to obtain $y = 18$. Substitute into the first equation: $2x + (-1)(18) = -10$, giving $x = 4$.

4. Check: $(2)(4) + (-1)(18) = -10$; $(4)(4) + (-1)(18) = -2$. Both constraints hold, so the pair is unique.

Problem 8

1. The doubled report adds no information because $2(x + y) = 2(8) = 16$. For the target prices, the difference is $5 - 3 = 2$.

2. Add the new condition $x - y = 2$ to $x + y = 8$. Adding these independent equations gives $2x = 10$, so $x = 5$.

3. Substitute into the total: $5 + y = 8$, giving $y = 3$. Both original reports and the new difference are satisfied, so the target prices are recovered.

Problem 9

1. The claim gives $5 + 6 = 11$ coins and $5(5) + 10(6) = 25 + 60 = 85$ cents. Both totals match; elimination verifies the unique coin counts.

2. The scaled equations are $5x + 5y = 55$ and $5x + 10y = 85$. Subtracting the second from the first gives $-5y = -30$.

3. Divide to obtain $y = 6$. Substitute into the first equation: $1x + (1)(6) = 11$, giving $x = 5$.

4. Check: $(1)(5) + (1)(6) = 11$; $(5)(5) + (10)(6) = 85$. Both constraints hold, so the pair is unique.

Problem 10

1. Eliminate x: multiply the first equation by 1 and the second by 1. The resulting x coefficients both equal 1.

2. The scaled equations are $1x + 1y = 10$ and $1x - 1y = 14$. Subtracting the second from the first gives $2y = -4$.

3. Divide to obtain $y = -2$. Substitute into the first equation: $1x + (1)(-2) = 10$, giving $x = 12$.

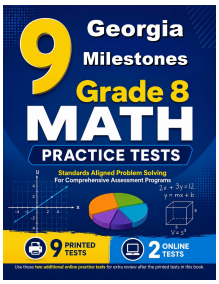
4. Check: $(1)(12) + (1)(-2) = 10$; $(1)(12) + (-1)(-2) = 14$. A negative dimension is not physically valid.

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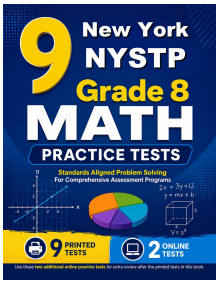
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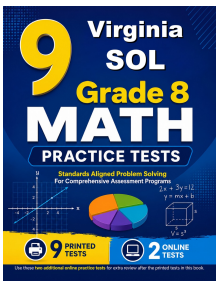
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