Solving Real Problems with Systems

Real Systems: Counts, Prices, and Measurements

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Define two variables with units and translate two independent facts into equations. A total count gives $x + y = N$; a value total may give $5x + 3y = V$. Solve both together and check count, value, and feasibility. Counts must be whole and nonnegative, while lengths and modeled prices may be fractional. Repeated information does not necessarily determine one unique answer.

Worked example

A show sells 12 tickets for $\$52$. Adult tickets cost $\$5$ and child tickets $\$3$. Find adult count x and child count y. $1x + 1y = 12$, $5x + 3y = 52$.

1. Eliminate x: multiply the first equation by 5 and the second by 1. The resulting x coefficients both equal 5.

2. The scaled equations are $5x + 5y = 60$ and $5x + 3y = 52$. Subtracting the second from the first gives $2y = 8$.

3. Divide to obtain $y = 4$. Substitute into the first equation: $1x + (1)(4) = 12$, giving $x = 8$.

4. Check: $(1)(8) + (1)(4) = 12$; $(5)(8) + (3)(4) = 52$. Both constraints hold, so the pair is unique.

Answer: $(x, y) = (8, 4)$.

Practice Problems

Show your thinking.

1. There are 15 bicycles and tricycles with 36 wheels altogether. Find bicycles x and tricycles y. $1x + 1y = 15$, $2x + 3y = 36$.

2. A jar contains 14 nickels and dimes worth 95 cents. Find nickels x and dimes y. $1x + 1y = 14$, $5x + 10y = 95$.

3. Two numbers sum to 31; the first exceeds the second by 7. Find the numbers x, y. $1x + 1y = 31$, $1x - 1y = 7$.

4. A rectangle has perimeter 46 centimeters. Length x is 5 centimeters more than width y. Find both dimensions. $1x + 1y = 23$, $1x - 1y = 5$.

5. Two notebooks and a pen cost $\$9$; one notebook and two pens cost $\$6$. Find notebook price x and pen price y. $2x + 1y = 9$, $1x + 2y = 6$.

6. A club has 20 student and adult members. Student dues are $\$4$, adult dues $\$7$, and total dues $\$104$. Find students x and adults y. $1x + 1y = 20$, $4x + 7y = 104$.

7. At a constant speed, a taxi bill is $\$13$ for 3 kilometers and $\$21$ for 7 kilometers. Let x be dollars per kilometer and y the fixed fee. Find both. $3x + 1y = 13$, $7x + 1y = 21$.

8. A supplier lists a board price x and bracket price y, both nonnegative real dollar amounts. One board plus two brackets costs $\$10$; twice that bundle costs $\$20$. Are prices unique? $1x + 2y = 10$, $2x + 4y = 20$.

9. One invoice says a board x plus a bracket y costs $\$8$. Another says two of each cost $\$19$. Can fixed prices satisfy both invoices? $1x + 1y = 8$, $2x + 2y = 19$.

10. Two runners have distances $d = 3t + 2$ and $d = 2t + 6$ kilometers. Find meeting time $x = t$ and distance $y = d$. $3x - 1y = -2$, $2x - 1y = -6$.

Answer Key

1. $(x, y) = (9, 6)$.
2. $(x, y) = (9, 5)$.
3. $(x, y) = (19, 12)$.
4. $(x, y) = (14, 9)$.
5. $(x, y) = (4, 1)$.
6. $(x, y) = (12, 8)$.
7. $(x, y) = (2, 7)$.
8. Infinitely many feasible solutions: $(x, y) = (10 - 2u, u)$, $0 \le u \le 5$.
9. No solution.
10. $(x, y) = (4, 14)$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Eliminate x: multiply the first equation by 2 and the second by 1. The resulting x coefficients both equal 2.
2. The scaled equations are $2x + 2y = 30$ and $2x + 3y = 36$. Subtracting the second from the first gives $-1y = -6$.
3. Divide to obtain $y = 6$. Substitute into the first equation: $1x + (1)(6) = 15$, giving $x = 9$.
4. Check: $(1)(9) + (1)(6) = 15$; $(2)(9) + (3)(6) = 36$. Both constraints hold, so the pair is unique.

Problem 2

1. Eliminate x: multiply the first equation by 5 and the second by 1. The resulting x coefficients both equal 5.
2. The scaled equations are $5x + 5y = 70$ and $5x + 10y = 95$. Subtracting the second from the first gives $-5y = -25$.
3. Divide to obtain $y = 5$. Substitute into the first equation: $1x + (1)(5) = 14$, giving $x = 9$.
4. Check: $(1)(9) + (1)(5) = 14$; $(5)(9) + (10)(5) = 95$. Both constraints hold, so the pair is unique.

Problem 3

1. Eliminate x: multiply the first equation by 1 and the second by 1. The resulting x coefficients both equal 1.
2. The scaled equations are $1x + 1y = 31$ and $1x - 1y = 7$. Subtracting the second from the first gives $2y = 24$.
3. Divide to obtain $y = 12$. Substitute into the first equation: $1x + (1)(12) = 31$, giving $x = 19$.
4. Check: $(1)(19) + (1)(12) = 31$; $(1)(19) + (-1)(12) = 7$. Both constraints hold, so the pair is unique.

Problem 4

1. Eliminate x: multiply the first equation by 1 and the second by 1. The resulting x coefficients both equal 1.
2. The scaled equations are $1x + 1y = 23$ and $1x - 1y = 5$. Subtracting the second from the first gives $2y = 18$.
3. Divide to obtain $y = 9$. Substitute into the first equation: $1x + (1)(9) = 23$, giving $x = 14$.
4. Check: $(1)(14) + (1)(9) = 23$; $(1)(14) + (-1)(9) = 5$. Both constraints hold, so the pair is unique.

Problem 5

1. Eliminate x: multiply the first equation by 1 and the second by 2. The resulting x coefficients both equal 2.
2. The scaled equations are $2x + 1y = 9$ and $2x + 4y = 12$. Subtracting the second from the first gives $-3y = -3$.
3. Divide to obtain $y = 1$. Substitute into the first equation: $2x + (1)(1) = 9$, giving $x = 4$.
4. Check: $(2)(4) + (1)(1) = 9$; $(1)(4) + (2)(1) = 6$. Both constraints hold, so the pair is unique.

Problem 6

1. Eliminate x: multiply the first equation by 4 and the second by 1. The resulting x coefficients both equal 4.
2. The scaled equations are $4x + 4y = 80$ and $4x + 7y = 104$. Subtracting the second from the first gives $-3y = -24$.
3. Divide to obtain $y = 8$. Substitute into the first equation: $1x + (1)(8) = 20$, giving $x = 12$.
4. Check: $(1)(12) + (1)(8) = 20$; $(4)(12) + (7)(8) = 104$. Both constraints hold, so the pair is unique.

Problem 7

1. Eliminate x: multiply the first equation by 7 and the second by 3. The resulting x coefficients both equal 21.
2. The scaled equations are $21x + 7y = 91$ and $21x + 3y = 63$. Subtracting the second from the first gives $4y = 28$.
3. Divide to obtain $y = 7$. Substitute into the first equation: $3x + (1)(7) = 13$, giving $x = 2$.
4. Check: $(3)(2) + (1)(7) = 13$; $(7)(2) + (1)(7) = 21$. Both constraints hold, so the pair is unique.

Problem 8

1. Eliminate x: multiply the first equation by 2 and the second by 1. The resulting x coefficients both equal 2.
2. The scaled equations are $2x + 4y = 20$ and $2x + 4y = 20$. Subtracting the second from the first gives $0y = 0$.
3. With $y = u$, $x = 10 - 2u$. Nonnegativity requires $u \ge 0$ and $10 - 2u \ge 0$, hence $0 \le u \le 5$. This interval gives infinitely many feasible pairs.

Problem 9

1. Eliminate x: multiply the first equation by 2 and the second by 1. The resulting x coefficients both equal 2.
2. The scaled equations are $2x + 2y = 16$ and $2x + 2y = 19$. Subtracting the second from the first gives $0y = -3$.
3. The result $0 = -3$ is false. No ordered pair can satisfy both equations, so the system has no solution.

Problem 10

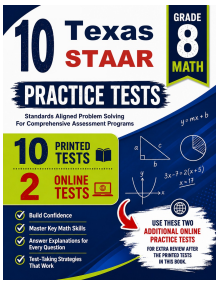
1. Eliminate x: multiply the first equation by 2 and the second by 3. The resulting x coefficients both equal 6.
2. The scaled equations are $6x - 2y = -4$ and $6x - 3y = -18$. Subtracting the second from the first gives $1y = 14$.
3. Divide to obtain $y = 14$. Substitute into the first equation: $3x + (-1)(14) = -2$, giving $x = 4$.
4. Check: $(3)(4) + (-1)(14) = -2$; $(2)(4) + (-1)(14) = -6$. Both constraints hold, so the pair is unique.

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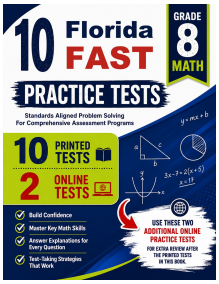
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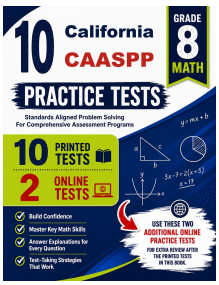
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