Solving Linear Inequalities

Inequalities: Budgets, Limits, and Whole Counts

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Use inequalities for limits rather than exact targets. At most means $\leq$; at least means $\geq$; more than means $>$. Translate fixed and variable costs before solving, and enforce nonnegative whole counts for indivisible items. If $n \leq 4.6$, the greatest whole count is 4, not 5. Check the chosen count and its next neighbor against the original constraint.

Worked example

A trip costs $\$12$ plus $\$5$ per rider. With at most $\$39$, find the greatest whole number of riders x.

1. The budget condition is $5x + 12 \leq 39$. Subtract the fixed fee: $5x \leq 27$, with x restricted to a nonnegative whole rider count.
2. Collect like terms while preserving order: $(5 - 0)x \leq 39 - (12)$. This simplifies to $5x \leq 27$.
3. Divide by 5: $x \leq \frac{27}{5}$. The positive divisor keeps the inequality direction unchanged. The greatest allowed integer is 5.
Answer: $x \leq \frac{27}{5}$. Greatest allowed whole count: 5.

Practice Problems

Show your thinking.

1. You have $\$50$; shipping costs $\$8$ and each book $\$6$. Find the greatest whole book count x within budget.

2. A club needs at least $\$95$, already has $\$20$, and gains $\$9$ per sale. Find the least whole sales count x.

3. A lift holds at most 450 kilograms. Equipment weighs 90 kilograms, and each package weighs 40. Find the greatest whole package count x.

4. A tank starts with 60 liters and loses 4 per minute. For how many elapsed minutes $x \geq 0$ is at least 20 liters left?

5. A game requires more than 100 points. You have 37 and earn 8 per round. Find the least whole extra-round count x.

6. A rectangle has width x and length $x + 2$ meters. Its perimeter is at most 28 meters. Find the greatest positive integer width.

7. Plan A costs $3x + 15$ dollars; plan B costs $5x + 3$ dollars. For which nonnegative use counts is A no more expensive than B?

8. Design the largest fixed fee for a $\$4$-per-visit plan if 6 visits must cost at most $\$31$.

9. A student needs an average of at least 80 on four tests. Scores are $72, 85, 79, x$. Find the least integer fourth score.

10. An account has $\$48$ and loses $\$7$ for each withdrawal. Its balance must stay strictly above $\$10$. Find the greatest whole withdrawal count x.

Answer Key

1. $x \leq 7$. Greatest allowed whole count: 7.
2. $x \geq \frac{25}{3}$. Least allowed whole count: 9.
3. $x \leq 9$. Greatest allowed whole count: 9.
4. $0 \leq x \leq 10$ minutes.
5. $x > \frac{63}{8}$. Least allowed whole count: 8.
6. $x \leq 6$. Greatest allowed whole count: 6.
7. $x \geq 6$.
8. Largest fixed fee $\$7$.
9. $x \geq 84$. Least allowed whole count: 84.
10. $x < \frac{38}{7}$. Greatest allowed whole count: 5.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Total cost is $6x + 8$, so the budget requires $6x + 8 \leq 50$. The book count must be a nonnegative integer.

2. Collect like terms while preserving order: $(6 - 0)x \leq 50 - (8)$. This simplifies to $6x \leq 42$.

3. Divide by 6: $x \leq 7$. The positive divisor keeps the inequality direction unchanged. The greatest allowed integer is 7.

Problem 2

1. Starting funds plus sales income gives $9x + 20 \geq 95$. This is a minimum target, so the comparison includes values above the target too.

2. Collect like terms while preserving order: $(9 - 0)x \geq 95 - (20)$. This simplifies to $9x \geq 75$.

3. Divide by 9: $x \geq \frac{25}{3}$. The positive divisor keeps the inequality direction unchanged. The least allowed integer is 9.

Problem 3

1. The total load is $40x + 90$ kilograms. Capacity imposes $40x + 90 \leq 450$, with no fractional packages permitted in the count.

2. Collect like terms while preserving order: $(40 - 0)x \leq 450 - (90)$. This simplifies to $40x \leq 360$.

3. Divide by 40: $x \leq 9$. The positive divisor keeps the inequality direction unchanged. The greatest allowed integer is 9.

Problem 4

1. The remaining volume is $60 - 4x$. Requiring at least 20 liters gives $-4x + 60 \geq 20$, together with the stated time restriction $x \geq 0$.

2. Collect like terms while preserving order: $(-4 - 0)x \geq 20 - (60)$. This simplifies to $-4x \geq -40$.

3. Divide by -4 and reverse: $x \leq 10$. With $x \geq 0$, the complete set is $0 \leq x \leq 10$ minutes.

Problem 5

1. The score after x extra rounds is $8x + 37$. More than 100 means the strict condition $8x + 37 > 100$, not an inclusive comparison.

2. Collect like terms while preserving order: $(8 - 0)x > 100 - (37)$. This simplifies to $8x > 63$.

3. Divide by 8: $x > \frac{63}{8}$. The positive divisor keeps the inequality direction unchanged. The least allowed integer is 8.

Problem 6

1. Perimeter is $2x + 2(x + 2) = 4x + 4$. The size limit gives $4x + 4 \leq 28$, while positive dimensions require $x > 0$.

2. Collect like terms while preserving order: $(4 - 0)x \leq 28 - (4)$. This simplifies to $4x \leq 24$.

3. Divide by 4: $x \leq 6$. The positive divisor keeps the inequality direction unchanged. The greatest allowed integer is 6.

Problem 7

1. Compare complete costs using $3x + 15 \leq 5x + 3$. The fixed fees and use rates both matter; comparing the rates alone would omit the initial charges.

2. Collect like terms while preserving order: $(3 - 5)x \leq 3 - (15)$. This simplifies to $-2x \leq -12$.

3. Divide by -2: $x \geq 6$. Reverse the inequality when dividing by a negative number.

Problem 8

1. Let b be the nonnegative fixed fee. Six visits cost $4(6) + b = 24 + b$, and the maximum-cost condition is $24 + b \leq 31$.

2. Subtract 24 to get $b \leq 7$. The largest possible fee is therefore 7 dollars, with equality allowed by the words at most.

3. Check the design: $4(6) + 7 = 31$ fits exactly. Any fee above 7 would raise the cost above 31, violating the stated limit.

Problem 9

1. The first three scores sum to $72 + 85 + 79 = 236$. The average requirement $(236 + x)/4 \geq 80$ becomes $236 + x \geq 320$.

2. Collect like terms while preserving order: $(1 - 0)x \geq 320 - (236)$. This simplifies to $1x \geq 84$.

3. Divide by 1: $x \geq 84$. The positive divisor keeps the inequality direction unchanged. The least allowed integer is 84.

Problem 10

1. After x withdrawals, the balance is $48 - 7x$. Staying strictly above 10 requires $-7x + 48 > 10$, with a nonnegative integer count.

2. Collect like terms while preserving order: $(-7 - 0)x > 10 - (48)$. This simplifies to $-7x > -38$.

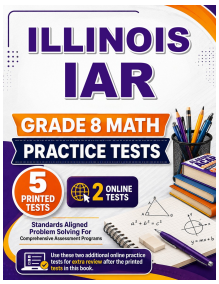
3. Divide by -7: $x < \frac{38}{7}$. Reverse the inequality when dividing by a negative number. The greatest allowed integer is 5.

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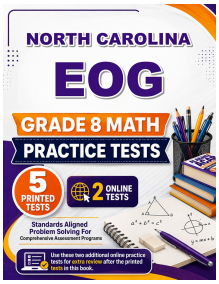
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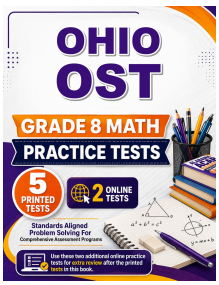
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