Solving Linear Inequalities

Inequalities: Repair Signs and Reverse Conditions

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Test a claimed inequality solution with a boundary value and a value on either side. A negative divisor reverses order: $-3x < 9$ gives $x > -3$, not $x < -3$. Distinguish a single satisfying value from the entire solution set. To design an inequality with a desired endpoint, substitute that endpoint first, then choose a strict or inclusive comparison to control membership.

Worked example

A student writes $-2x < 8 \Rightarrow x < -4$. Correct the result.

1. The proposed direction fails a check: $x = 0$ gives $0 < 8$, true, yet $0 < -4$ is false. Division by -2 must reverse the sign.

2. Collect like terms while preserving order: $(-2 - 0)x < 8 - (0)$. This simplifies to $-2x < 8$.

3. Divide by -2: $x > -4$. Reverse the inequality when dividing by a negative number.

Answer: $x > -4$.

Practice Problems

Show your thinking.

1. Repair $-5x \geq 15 \Rightarrow x \geq -3$.

2. A student changes $2x + 7 > 11$ into $2x > 18$. Correct and solve.

3. Choose c so $3x + c \leq 14$ has solution set $x \leq 4$.

4. Design an inequality with coefficient -2 and solution set $x > 3$.

5. Are $x < 5$ and $x \leq 5$ equivalent? Give a numerical counterexample.

6. Solve $6 - 3x > 2x + 1$ and explain which side of the endpoint works.

7. A student gives $x = 2$ as the complete answer to $x + 1 < 5$. Explain and state the full set.

8. Classify the solution set of $2(x + 3) < 2x + 9$.

9. Classify the solution set of $3x + 4 \leq 3x - 2$.

10. Find the greatest integer satisfying $2x + 1 < 10$, and explain why rounding to 5 fails.

Answer Key

1. $x \leq -3$.
2. $x > 2$.
3. $c = 2$.
4. One choice is $-2x < -6$.
5. No; $x = 5$ satisfies only $x \leq 5$.
6. $x < 1$.
7. $x < 4$; 2 is only one solution.
8. All real numbers.
9. No solution.
10. $x = 4$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The divisor is -5, not positive 5. For example, $x = 0$ gives $0 \geq 15$, false, so the proposed rightward solution set cannot be correct.

2. Collect like terms while preserving order: $(-5 - 0)x \geq 15 - (0)$. This simplifies to $-5x \geq 15$.

3. Divide by -5: $x \leq -3$. Reverse the inequality when dividing by a negative number.

Problem 2

1. Remove $+7$ by subtracting 7: $2x > 11 - 7 = 4$. Adding only to the right side would not preserve the solution set.

2. Collect like terms while preserving order: $(2 - 0)x > 11 - (7)$. This simplifies to $2x > 4$.

3. Divide by 2: $x > 2$. The positive divisor keeps the inequality direction unchanged.

Problem 3

1. The desired endpoint $x = 4$ must give equality. Substitute it: $3(4) + c = 14$, or $12 + c = 14$.

2. Solve for the constant: $c = 14 - 12 = 2$. The completed inequality is $3x + 2 \leq 14$, which reduces to $3x \leq 12$.

3. Dividing by positive 3 gives exactly $x \leq 4$. The endpoint is included because $3(4) + 2 = 14$ satisfies the inclusive comparison.

Problem 4

1. Start with the desired set $x > 3$. Multiply both sides by -2 and reverse the comparison, giving $-2x < -6$.

2. Solving the designed inequality by division by -2 reverses it again: $x > 3$. This recovers the intended entire solution set.

3. Check $x = 4$: $-8 < -6$ is true. Check the excluded endpoint $x = 3$: $-6 < -6$ is false. Both checks match the design.

Problem 5

1. Test the shared endpoint $x = 5$. The first comparison becomes $5 < 5$, which is false because a strict inequality excludes equality.

2. The second becomes $5 \leq 5$, which is true. Thus one set contains 5 while the other does not.

3. The sets are not equivalent even though every value below 5 belongs to both. Their difference is exactly the included endpoint 5.

Problem 6

1. Subtract $2x$ from both sides: $6 - 5x > 1$. The variable coefficient becomes -5, so the final division will reverse the comparison.

2. Collect like terms while preserving order: $(-3 - 2)x > 1 - (6)$. This simplifies to $-5x > -5$.

3. Divide by -5: $x < 1$. Reverse the inequality when dividing by a negative number.

Problem 7

1. The proposed value does work because $2 + 1 = 3 < 5$. However, one successful substitution does not identify every value satisfying an inequality.

2. Subtract 1 from both sides of $x + 1 < 5$ to obtain $x < 4$. This describes all real solutions, not just integers.

3. For example, $3 + 1 = 4 < 5$ and $-1 + 1 = 0 < 5$ also work. Therefore the complete answer is $x < 4$, not only $x = 2$.

Problem 8

1. Distribute the left side: $2x + 6 < 2x + 9$. Subtract $2x$ from each side, leaving the purely numerical comparison $6 < 9$.

2. The statement $6 < 9$ is always true and contains no variable. It places no restriction on any real value of x.

3. Therefore every real number is a solution. For example, $x = 0$ gives $6 < 9$, and $x = -4$ gives $-2 < 1$, both true.

Problem 9

1. Subtract $3x$ from both sides of the inequality. The remaining comparison is $4 \leq -2$, with no variable term left.

2. The statement $4 \leq -2$ is false because $4 > -2$. No choice of input can change this constant contradiction.

3. Therefore the solution set is empty. For example, $x = 0$ already gives $4 \leq -2$, and cancellation shows every other input fails as well.

Problem 10

1. Subtract 1 and divide by 2: $2x < 9$, so $x < 4.5$. The task asks for an allowed integer, not a rounded endpoint.

2. The greatest integer below 4.5 is 4. Checking it gives $2(4) + 1 = 9 < 10$, which is true.

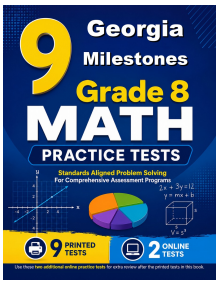
3. Rounding 4.5 to 5 would fail because $2(5) + 1 = 11 \not< 10$. Therefore the greatest allowed integer is exactly 4.

TESTINAR

Keep Building Grade 8 Math Skills

Choose the practice book matched to your state assessment and turn worksheet practice into a complete test-prep plan.

All Grade 8 books



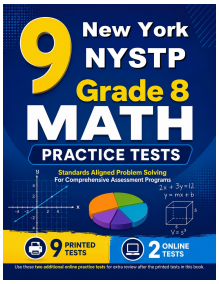
GEORGIA MILESTONES

9 Full-Length Math Practice Tests

Build Grade 8 accuracy, confidence, and test stamina with 9 printable practice tests.

Printable eBook • Instant PDF access

View eBook



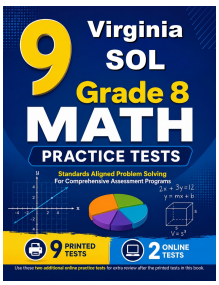
NEW YORK NYSTP

9 Full-Length Math Practice Tests

Build Grade 8 accuracy, confidence, and test stamina with 9 printable practice tests.

Printable eBook • Instant PDF access

View eBook



VIRGINIA SOL

9 Full-Length Math Practice Tests

Build Grade 8 accuracy, confidence, and test stamina with 9 printable practice tests.

Printable eBook • Instant PDF access

View eBook

Need a different state?
Testinar has Grade 8 math practice books for state assessments across the country.

TESTINAR.COM/GRADES/8