Solving Linear Equations in One Variable

Linear Equations: Repair and Reconstruct

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

An algebra step is valid only if it preserves equality. In $-2(x-3)$, distribution gives $-2x+6$, not $-2x-6$. When variables cancel, inspect the remaining numerical statement before dividing. Work backward by substituting a desired solution into an equation with a missing coefficient or constant. Verify the repaired equation in the original form, not only in a later step.

Worked example

A student writes $-2(x-3)=10 \Rightarrow -2x-6=10$. Correct and solve.

1. The product $(-2)(-3)=6$ is positive, so the correct distribution is $-2x+6=10$. The student lost the sign of the second factor.

2. Collecting variable terms on the left and constants on the right gives $(-2-0)x=10-(6)$, or $-2x=4$.

3. Divide by -2: $x=-2$. In the reduced equation, both sides equal 10, confirming the solution.

Answer: $x=-2$.

Practice Problems

Show your thinking.

1. A student solves $3x+5=17$ as $3x=22$. Correct the step and finish.

2. Repair $4(x+2)=20 \Rightarrow 4x+2=20$, then solve.

3. Choose k so that $2x+k=19$ has solution $x=6$.

4. Find a so $ax-4=11$ is solved by $x=3$.

5. Explain why $5(x-1)=5x-5$ does not have only the solution $x=0$.

6. A student divides by 0 after simplifying $7x+2=7x-1$. Classify the equation correctly.

7. Fill the blank so $3(x+4)=3x+\square$ has infinitely many solutions.

8. Choose a constant c so $2x+5=2x+c$ has no solution. Explain your choice.

9. Which step first removes fractions from $\frac{x}{4}+\frac{1}{2}=3$? Use it and solve.

10. Design an equation with x on both sides and unique solution $x=-3$. Show a check.

Answer Key

1. $x = 4$.
2. $x = 3$.
3. $k = 7$.
4. $a = 5$.
5. Infinitely many solutions.
6. No solution.
7. $\square = 12$.
8. One choice is $c = 6$.
9. $x = 10$.
10. One choice is $2x + 1 = x - 2$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. To remove $+5$, subtract 5 from both sides: $3x = 17 - 5 = 12$. Adding 5 to only the right side changes the equation.

2. Collecting variable terms on the left and constants on the right gives $(3 - 0)x = 17 - (5)$, or $3x = 12$.

3. Divide by 3: $x = 4$. In the reduced equation, both sides equal 17, confirming the solution.

Problem 2

1. The outside factor multiplies both terms: $4(x + 2) = 4x + 8$, not $4x + 2$. The corrected equivalent equation is $4x + 8 = 20$.

2. Collecting variable terms on the left and constants on the right gives $(4 - 0)x = 20 - (8)$, or $4x = 12$.

3. Divide by 4: $x = 3$. In the reduced equation, both sides equal 20, confirming the solution.

Problem 3

1. Substitute the desired solution into the equation: $2(6) + k = 19$. This makes the missing constant the only unknown in the numerical equality.

2. Calculate $12 + k = 19$. Subtract 12 from both sides to find $k = 19 - 12 = 7$.

3. The completed equation is $2x + 7 = 19$. Checking $x = 6$ gives $2(6) + 7 = 12 + 7 = 19$, exactly as required.

Problem 4

1. Replace x with 3 in the required equation: $3a - 4 = 11$. The target solution becomes a condition on the coefficient a.

2. Add 4 to both sides to obtain $3a = 15$. Divide by 3, giving $a = 5$.

3. The resulting equation is $5x - 4 = 11$. At $x = 3$, its left side is $5(3) - 4 = 15 - 4 = 11$, confirming the design.

Problem 5

1. Distribute the left side to obtain $5x - 5 = 5x - 5$. The expressions agree for all inputs; substituting zero is only one possible check.

2. Collecting terms gives $(5 - 5)x = -5 - (-5)$, which simplifies to $0 = 0$ after the variable terms cancel.

3. The statement $0 = 0$ is true for every real x. Therefore every real number satisfies the original equation; there are infinitely many solutions.

Problem 6

1. Subtracting $7x$ from both sides gives $2 = -1$. The variable coefficient cancels, so dividing by that zero coefficient is not a valid operation.

2. Collecting terms gives $(7 - 7)x = -1 - (2)$, which simplifies to $0 = -3$ after the variable terms cancel.

3. The statement $0 = -3$ is false, regardless of x. Therefore no real number can satisfy the original equation, and the solution set is empty.

Problem 7

1. Distribute the left side: $3(x + 4) = 3x + 12$. For an identity, the right side must be exactly the same expression.

2. Subtracting $3x$ leaves $12 = \square$, so choose 12. With this choice the reduced equation becomes the true statement $12 = 12$.

3. Therefore every real x works. For instance, $x = 2$ gives $3(2 + 4) = 18$ and $3(2) + 12 = 18$, confirming one example of the identity.

Problem 8

1. The variable coefficients on the two sides already match. Subtracting $2x$ will leave $5 = c$, independent of the value of x.

2. Choose $c = 6$, a constant different from 5. The reduced statement is then $5 = 6$, which is false.

3. Thus $2x + 5 = 2x + 6$ has no solution. Any real constant other than 5 would also create a contradiction after the variable terms cancel.

Problem 9

1. Multiply every term by 4: $4(\frac{x}{4}) + 4(\frac{1}{2}) = 4(3)$. This gives $x + 2 = 12$, not $x + \frac{1}{2} = 12$.

2. Collecting variable terms on the left and constants on the right gives $(1 - 0)x = 12 - (2)$, or $1x = 10$.

3. Divide by 1: $x = 10$. In the reduced equation, both sides equal 12, confirming the solution.

Problem 10

1. Choose different variable coefficients, 2 and 1. To make $x = -3$ work, use constants 1 and -2: $2x + 1 = x - 2$.

2. Solving the proposed equation gives $x + 1 = -2$ after subtracting x, then $x = -3$ after subtracting 1. The remaining coefficient is nonzero.

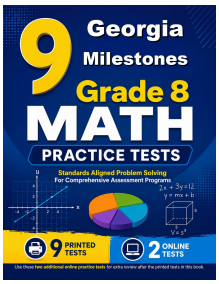
3. Check both original sides: $2(-3) + 1 = -5$ and $-3 - 2 = -5$. Thus the designed equation has exactly the required unique solution.

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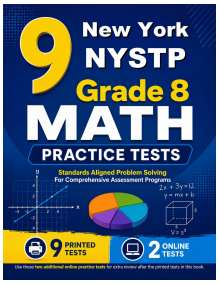
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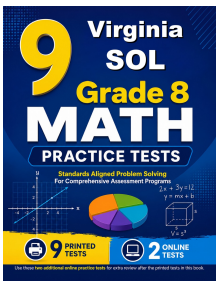
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