Slope as Rate of Change

Slope: Rates in Decisions and Designs

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

For measured data, slope is change in output divided by elapsed input, with units attached. If a balance falls from $\$20$ to $\$8$ in 3 days, the rate is $(8-20)/3=-4$ dollars per day. Use the signed rate to predict changes, compare equal units, and check whether a proposed design meets a maximum or minimum rate.

Worked example

A cyclist travels 6 kilometers by hour 1 and 18 kilometers by hour 3. Find the constant speed.

1. The distance change is $18-6=12$ kilometers. The elapsed time is $3-1=2$ hours, not the final clock reading of 3 hours.

2. Compute the slope $m=12/2=6$ kilometers per hour. This positive rate measures distance gained during each elapsed hour.

3. Check by adding two hours of travel: $6+6(2)=18$ kilometers. The calculated speed agrees with both measurements.

Answer: 6 kilometers per hour.

Practice Problems

Show your thinking.

1. A candle is 24 centimeters tall at hour 0 and 15 centimeters at hour 3. Find its signed rate and height at hour 5.

2. A ramp rises 0.6 meter over a horizontal run of 5 meters. Is its slope at most 0.125?

3. A tank loses water at 4 liters per minute. Design two readings 3 minutes apart, beginning with 25 liters at minute 2.

4. Plan A gains 18 points in 6 rounds; plan B gains 14 points in 4 rounds. Which has the greater average rate?

5. A freezer cools from 8 degrees Celsius to -4 degrees Celsius in 6 hours. Find its average signed rate.

6. A delivery fee changes from $\$11$ for 2 kilometers to $\$20$ for 5 kilometers. Find the extra charge per kilometer.

7. A walking route drops 45 meters over 300 horizontal meters. Find the signed slope as a decimal and a percentage.

8. Design a ramp with slope $\frac{1}{12}$ and a rise of 0.5 meter. Find the required horizontal run.

9. A battery falls from 90% to 66% in 2 hours. At this constant rate, how many more hours until 30%?

10. An elevator moves from floor -2 to floor 7 in 18 seconds at a constant rate. Find the rate and time for another 4 floors up.

Answer Key

1. -3 centimeters per hour; 9 centimeters.
2. Yes; $m = 0.12 \le 0.125$.
3. $(2, 25)$ and $(5, 13)$, with time in minutes.
4. Plan B: 3.5 versus 3 points per round.
5. -2 degrees Celsius per hour.
6. $\$3$ per kilometer.
7. $-0.15 = -15\%$.
8. 6 meters.
9. 3 more hours.
10. 0.5 floor per second; 8 seconds.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The height change is $15 - 24 = -9$ over $3 - 0 = 3$ hours. Therefore the rate is $m = -9/3 = -3$ centimeters per hour.

2. From hour 3 to hour 5 is 2 more hours. The change is $(-3)(2) = -6$ centimeters.

3. The predicted height is $15 - 6 = 9$ centimeters. This assumes the same burn rate continues and the candle has not yet burned out.

Problem 2

1. Use the vertical rise 0.6 meter and horizontal run 5 meters. The slope is $m = 0.6/5 = 0.12$.

2. Compare in the same decimal format: $0.120 < 0.125$. No rounding is needed because $0.6/5$ terminates exactly.

3. The ramp meets the stated maximum slope. At a run of 5, the allowed rise would be $0.125(5) = 0.625$ meter, above 0.6.

Problem 3

1. The signed rate is -4 liters per minute. Over 3 minutes, the change is $(-4)(3) = -12$ liters.

2. The second time is $2 + 3 = 5$ minutes and the second amount is $25 - 12 = 13$ liters. Both amounts are nonnegative.

3. Check the slope of the readings: $(13 - 25)/(5 - 2) = -12/3 = -4$. Thus the designed readings match the specified loss rate.

Problem 4

1. For plan A, divide the point change by the round change: $m_A = 18/6 = 3$ points per round.

2. For plan B, calculate $m_B = 14/4 = 3.5$ points per round. Both quantities now use identical units and can be compared directly.

3. Since $3.5 > 3$, plan B has the greater average rate by $3.5 - 3 = 0.5$ point per round, despite gaining fewer total points.

Problem 5

1. The temperature change crosses zero: $-4 - 8 = -12$ degrees Celsius. The elapsed time is positive 6 hours.

2. Calculate $m = -12/6 = -2$ degrees Celsius per hour. A negative rate is appropriate because the temperature is falling.

3. Check the total change using the rate: $(-2)(6) = -12$. Adding this change to 8 gives $8 - 12 = -4$, the recorded final temperature.

Problem 6

1. The fee difference is $20 - 11 = 9$ dollars, and the distance difference is $5 - 2 = 3$ kilometers. Starting fees cancel in the difference.

2. The slope is $m = 9/3 = 3$ dollars per kilometer. Dividing total fee 20 by distance 5 would incorrectly include the initial charge.

3. Three additional kilometers add $3(3) = 9$ dollars. The check $11 + 9 = 20$ confirms the extra charge is $\$3$ per kilometer.

Problem 7

1. The vertical change is -45 meters because the route drops. The horizontal change is 300 meters in the direction of travel.

2. Compute $m = -45/300 = -0.15$. Multiply by 100% to express the same signed rate as -15%.

3. The negative sign indicates descent. For each 100 horizontal meters, the route loses 15 vertical meters; the downward grade magnitude is 15%.

Problem 8

1. Slope equals rise divided by run, so $\frac{1}{12} = 0.5/r$. Equivalently, the rise is one twelfth of the horizontal run.

2. Multiply by $12r$: $r = 12(0.5) = 6$. The required run is positive because both the rise and the slope are positive.

3. Check $0.5/6 = 1/12$. Therefore a horizontal run of 6 meters gives exactly the specified slope for a half-meter rise.

Problem 9

1. The charge change is $66 - 90 = -24$ percentage points over 2 hours. The signed rate is $-24/2 = -12$ percentage points per hour.

2. From 66% to 30%, the required change is $30 - 66 = -36$ percentage points. Solve $-36 = -12t$ for the additional time.

3. Dividing gives $t = 3$ hours. Check $66 - 12(3) = 30$; this is additional time after the second reading, not time from the beginning.

Problem 10

1. The floor change is $7 - (-2) = 9$, including the floors below zero. Divide by 18 seconds to obtain $m = 9/18 = 0.5$.

2. For a further rise of 4 floors, solve $4 = 0.5t$. This uses the same upward rate and assumes no intervening stop.

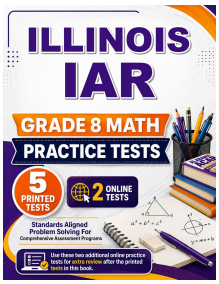
3. Dividing by 0.5 gives $t = 8$ seconds. The check $0.5(8) = 4$ floors confirms the additional travel time.

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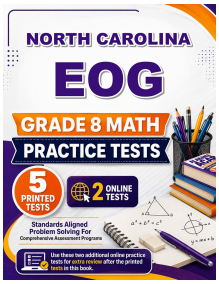
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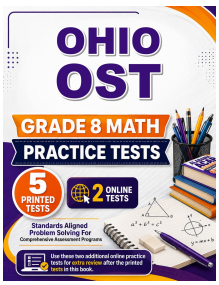
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