Slope as Rate of Change

Slope: Explain, Repair, and Work Backward

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Slope is a signed ratio, not merely a count of squares. Reversing both point differences leaves m unchanged: $6/3 = (-6)/(-3) = 2$. Reversing only one changes the sign incorrectly. Use $\Delta y = m\Delta x$ to reconstruct coordinates. A vertical line has zero run and undefined slope; a horizontal line has zero rise and slope 0.

Worked example

A student uses $(1, 5), (4, -1)$ and writes $m = (5 - (-1))/(4 - 1) = 2$. Correct the work.

1. The numerator $5 - (-1) = 6$ goes from the second point to the first. Its matching denominator must be $1 - 4 = -3$.
2. Calculate $m = 6/(-3) = -2$. Alternatively, use the opposite order consistently: $(-1 - 5)/(4 - 1) = -6/3 = -2$.
3. The output falls from 5 to -1 as the input grows from 1 to 4. A negative slope is therefore required.
Answer: $m = -2$; the subtraction orders were mixed.

Practice Problems

Show your thinking.

1. Lee says the slope through $(2, 1), (6, 3)$ is 2 because run divided by rise is $4/2$. Correct Lee.

2. Find a if $(1, 2)$ and $(5, a)$ lie on a line of slope 3.

3. Find b if $(-2, 7)$ and $(b, 1)$ lie on a line of slope -2.

4. Are slopes through $(0, 0), (2, 6)$ and $(2, 6), (4, 9)$ equal? Use the result to test collinearity.

5. Explain why reversing both differences for $(1, 4), (3, 10)$ does not change the slope. Compute both ratios.

6. A student says the line through $(-1, 4), (5, 4)$ has undefined slope. Identify and correct the confusion.

7. Give a second integer-coordinate point besides $(2, 3)$ on a line with slope $-\frac{2}{3}$. Verify it.

8. Two lines have slopes -4 and $-\frac{1}{2}$. Which has the larger slope, and which falls faster per unit run?

9. Complete $(x, y) = (0, 8), (2, 5), (6, c)$ so the rate is constant.

10. A line through $(4, 1)$ has undefined slope. Can $(5, 1)$ be on it? Explain numerically.

Answer Key

1. $m = \frac{1}{2}$.
2. $a = 14$.
3. $b = 1$.
4. No; slopes 3 and $\frac{3}{2}$; not collinear.
5. $6/2 = (-6)/(-2) = 3$.
6. Slope 0, not undefined.
7. One choice is $(5, 1)$.
8. $-\frac{1}{2}$ is larger; slope -4 falls faster.
9. $c = -1$.
10. No; the line is $x = 4$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The rise is $3 - 1 = 2$ and the run is $6 - 2 = 4$. Lee computed the reciprocal by dividing run by rise.

2. The correct slope is $m = \Delta y/\Delta x = 2/4 = \frac{1}{2}$. This ratio uses output change per input change.

3. Check with the total run: $\frac{1}{2}(4) = 2$, the actual rise. Lee's proposed rate would predict a rise of 8, not 2.

Problem 2

1. The horizontal change is $5 - 1 = 4$. A slope of 3 requires vertical change $\Delta y = 3(4) = 12$.

2. The output must increase from 2 by 12, so $a = 2 + 12 = 14$. Equivalently, $(a - 2)/4 = 3$.

3. Check the reconstructed point: $(14 - 2)/(5 - 1) = 12/4 = 3$. Thus $(5, 14)$ gives the required slope exactly.

Problem 3

1. The vertical change is $1 - 7 = -6$. With slope -2, solve $-6 = -2\Delta x$ to obtain $\Delta x = 3$.

2. The first input is -2, so the new input is $b = -2 + 3 = 1$. A positive run matches the negative vertical change.

3. Check $m = (1 - 7)/(1 - (-2)) = -6/3 = -2$. Therefore $b = 1$ is the missing coordinate.

Problem 4

1. Using the first two points, $m_1 = (6 - 0)/(2 - 0) = 6/2 = 3$. This rate describes only the first segment.

2. Using the last two points, $m_2 = (9 - 6)/(4 - 2) = 3/2 = \frac{3}{2}$. The horizontal intervals are equal but the rises differ.

3. Since $3 \neq \frac{3}{2}$, one straight nonvertical line cannot contain all three points. Therefore the points are not collinear.

Problem 5

1. In forward order, $\Delta y = 10 - 4 = 6$ and $\Delta x = 3 - 1 = 2$, giving slope $6/2 = 3$.

2. In reverse order, $\Delta y = 4 - 10 = -6$ and $\Delta x = 1 - 3 = -2$, giving slope $(-6)/(-2) = 3$.

3. Both numerator and denominator changed signs, so their quotient stayed positive and unchanged. The line therefore has the same slope 3 in either order.

Problem 6

1. The rise is $4 - 4 = 0$, while the run is $5 - (-1) = 6$. The denominator is not zero in this calculation.

2. Compute $m = 0/6 = 0$. Undefined slope would require division by zero, which occurs with a vertical line instead.

3. These points lie on $y = 4$, a horizontal line. It has no output change for any nonzero horizontal change, so its slope is zero.

Problem 7

1. The slope $-\frac{2}{3}$ allows a run of 3 and rise of -2. Starting at $(2, 3)$, add these signed changes.

2. The new point is $(2 + 3, 3 - 2) = (5, 1)$. Both coordinates are integers and the new point differs from the starting point.

3. Check $m = (1 - 3)/(5 - 2) = -2/3$. Thus $(5, 1)$ is one valid answer; other points with the same ratio also work.

Problem 8

1. On the number line, $-\frac{1}{2} = -0.5$ lies to the right of -4. Therefore $-\frac{1}{2} > -4$ is the numerical comparison.

2. For a run of 1, the changes are $(-4)(1) = -4$ and $(-\frac{1}{2})(1) = -\frac{1}{2}$. Both lines decrease.

3. The magnitudes satisfy $4 > 0.5$, so slope -4 gives the faster fall. A larger signed slope need not mean a larger decrease.

Problem 9

1. The first interval has slope $(5 - 8)/(2 - 0) = -3/2$. The next horizontal interval is $6 - 2 = 4$, not 2.

2. The next vertical change must be $(-\frac{3}{2})(4) = -6$. Add it to the last output: $c = 5 - 6 = -1$.

3. Check the second rate: $(-1 - 5)/(6 - 2) = -6/4 = -\frac{3}{2}$. Both intervals agree, so the completed table has a constant rate.

Problem 10

1. An undefined slope through $(4, 1)$ means every point has input 4. Its equation is $x = 4$, regardless of the output.

2. The proposed point has input 5, and $5 \neq 4$. It therefore fails the equation of the required vertical line.

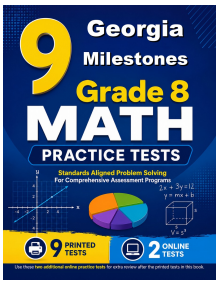
3. Indeed, the two given points together would have slope $(1 - 1)/(5 - 4) = 0/1 = 0$, which is horizontal rather than undefined.

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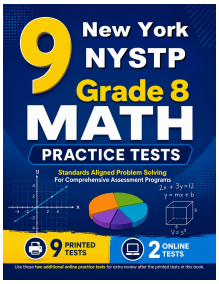
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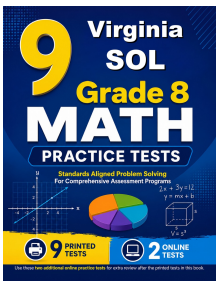
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