Slope as Rate of Change

Slope: Differences and Rate Models

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Slope measures vertical change per horizontal change: $m = \frac{y_2 - y_1}{x_2 - x_1}$. Keep the same point order in both differences. Between $(1, 2)$ and $(4, 8)$, $m = 6/3 = 2$. A horizontal line has slope 0; a vertical line has undefined slope because its run is 0. Include output units per input unit when interpreting a rate.

Worked example

Find the slope through $(-2, 5)$ and $(4, -1)$.

1. Using the second point minus the first, the vertical change is $-1 - 5 = -6$. The horizontal change is $4 - (-2) = 6$.

2. Substitute these differences into the slope formula: $m = \frac{-1-5}{4-(-2)} = \frac{-6}{6} = -1$. The denominator is nonzero.

3. The negative result means y falls by 1 for each increase of 1 in x. Thus the slope is -1.

Answer: $m = -1$.

Practice Problems

Show your thinking.

1. Find the slope through $(1, 3)$ and $(5, 11)$.

2. Find the slope through $(-3, 4)$ and $(3, -5)$.

3. A line rises 6 units while moving 9 units right. Find its slope in simplest form.

4. Find the slope through $(-4, 7)$ and $(2, 7)$. Classify the line.

5. Find the slope through $(3, -2)$ and $(3, 5)$. Classify the line.

6. The table has $(x, y) = (0, 9), (2, 5), (5, -1)$. Is the rate constant? Find it.

7. Find the slope through $(\frac{1}{2}, 1)$ and $(\frac{5}{2}, 4)$.

8. A line has slope $-\frac{3}{4}$. Find its vertical change for a horizontal change of 8.

9. Compare slopes through $A : (0, 1), (2, 7)$ and $B : (1, 8), (5, 12)$. Which increases faster?

10. A tank contains 18 liters at minute 1 and 10 liters at minute 5. Find and interpret the constant rate.

Answer Key

1. $m = 2$.
2. $m = -\frac{3}{2}$.
3. $m = \frac{2}{3}$.
4. $m = 0$; horizontal.
5. Undefined slope; vertical line $x = 3$.
6. Yes; $m = -2$.
7. $m = \frac{3}{2}$.
8. $\Delta y = -6$.
9. $m_A = 3$, $m_B = 1$; A increases faster.
10. -2 liters per minute; losing 2 liters each minute.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The vertical change is $11 - 3 = 8$ and the horizontal change is $5 - 1 = 4$, using the same point order.

2. Calculate $m = \frac{11-3}{5-1} = \frac{8}{4} = 2$. Both changes are positive, so the slope is positive.

3. A run of 4 gives a rise of 8. Dividing both by 4 shows a rise of 2 per unit run.

Problem 2

1. Calculate the output difference $-5 - 4 = -9$ and the input difference $3 - (-3) = 6$. The run is positive while the rise is negative.

2. Substitute into $m = \Delta y / \Delta x$: $m = -9/6 = -\frac{3}{2}$. Divide numerator and denominator by 3 to simplify.

3. The line drops 3 units for every 2 units to the right. This matches the negative slope $-\frac{3}{2}$.

Problem 3

1. Moving right gives a positive run of 9; rising gives a positive rise of 6. The signed changes are 9 and 6.

2. Compute slope as rise divided by run: $m = 6/9$. Cancel the common factor 3 to obtain $m = 2/3$.

3. The simplified slope means a rise of 2 for each run of 3. It is positive because both original changes were positive.

Problem 4

1. The two outputs are both 7, so the vertical change is $7 - 7 = 0$. The horizontal change is $2 - (-4) = 6$.

2. Substitute into the formula: $m = 0/6 = 0$. A zero numerator with a nonzero denominator gives a defined slope of zero.

3. The equation is $y = 7$, because the output stays fixed. Therefore the line is horizontal, not vertical.

Problem 5

1. The output difference is $5 - (-2) = 7$. Both inputs are 3, so the input difference is $3 - 3 = 0$.

2. The slope expression would be $7/0$. Division by zero is undefined, so this line has no numerical slope.

3. Both points satisfy $x = 3$, giving a vertical line. Undefined slope is not the same as the zero slope of a horizontal line.

Problem 6

1. Between the first two rows, $m = (5 - 9)/(2 - 0) = -4/2 = -2$. The unequal table intervals require comparing ratios, not output changes alone.

2. Between the last two rows, $m = (-1 - 5)/(5 - 2) = -6/3 = -2$. Both intervals give exactly the same rate.

3. Therefore the rate is constant at -2 output units per input unit. The table lies on the line $y = -2x + 9$.

Problem 7

1. The vertical difference is $4 - 1 = 3$. The horizontal difference is $\frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$, so the run is not fractional.

2. Substitute into the slope formula: $m = \frac{4-1}{5/2-1/2} = \frac{3}{2}$. No rounding is needed for this exact ratio.

3. The line rises 3 for a run of 2, or 1.5 per unit run. Both forms describe the same slope.

Problem 8

1. The rate equation is $\Delta y = m\Delta x$. Substitute the given slope and horizontal change to get $\Delta y = (-\frac{3}{4})(8)$.

2. Simplify $(-3)(8)/4 = -24/4 = -6$. The positive horizontal change combines with a negative slope to give a negative vertical change.

3. Therefore moving 8 units right moves 6 units down. The signed vertical change is -6, not positive 6.

Problem 9

1. For line A, calculate $m_A = (7 - 1)/(2 - 0) = 6/2 = 3$. Its output increases by 3 for each input unit.

2. For line B, calculate $m_B = (12 - 8)/(5 - 1) = 4/4 = 1$. Its larger starting output does not determine its rate.

3. Compare $3 > 1$. Thus A increases faster even though the listed outputs on B begin at a greater value.

Problem 10

1. The volume change is $10 - 18 = -8$ liters. The elapsed time is $5 - 1 = 4$ minutes, so use these differences in matching order.

2. The slope is $m = -8/4 = -2$ liters per minute. The negative sign records a decrease in the amount of water.

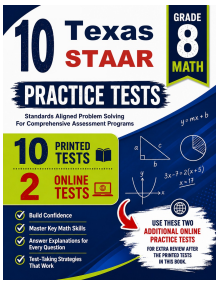
3. The tank loses 2 liters each minute over this interval. A check is $18 + (-2)(4) = 10$ liters at minute 5.

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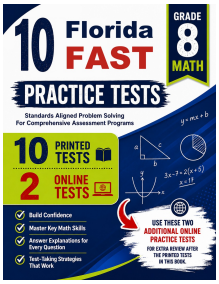
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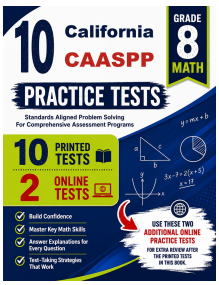
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