Slope and the Equations of a Line

Line Equations: Fees, Measurements, and Design

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

A linear application often combines a starting amount b and constant rate m in $y = mx + b$. A fee of $\$5$ plus $\$2$ per hour is $y = 2x + 5$; unlike $y = 2x$, it is not proportional. Use the context to restrict inputs, keep units consistent, and substitute known measurements before predicting an output or solving for a target input.

Worked example

A bike rental costs $\$4$ initially plus $\$3$ per hour. Write cost y for x hours.

1. The hourly charge is the slope $m = 3$ dollars per hour. At zero hours the initial charge is 4, giving the point $(0, 4)$.

2. Substitute the known point into $y = mx + b$: $4 = (3)(0) + b$. Since the product is 0, $b = 4$.

3. The equation is $y = 3x + 4$. Check the known point: $(3)(0) + (4) = 4$, so the line has the required point and slope.

Answer: $y = 3x + 4$.

Practice Problems

Show your thinking.

1. A taxi charges $\$6$ initially and $\$2$ per kilometer. Write cost C using distance x; use $y = C$.

2. A tank starts with 40 liters and drains 3 liters per minute. Write $V(t)$ and find $V(8)$.

3. A print shop charges $\$11$ for 2 posters and $\$23$ for 6 posters. Find its linear cost model y for x posters.

4. A plant is modeled by $h = 1.5w + 7$ centimeters after w weeks. Interpret 7 and find when $h = 19$.

5. Design a savings model starting at $\$12$ and reaching $\$42$ after 5 weeks with equal weekly deposits.

6. Plan A costs $A = 4h + 8$ dollars and plan B costs $B = 6h + 2$ dollars. Compare their starting fees and hourly rates; find both costs at $h = 2$.

7. A temperature scale uses $F = \frac{9}{5}C + b$ and reads $F = 32$ at $C = 0$. Find the model; use $x = C, y = F$.

8. A fundraiser has $\$25$ and gains $\$7$ per ticket sold. Write a model and find the balance after 9 tickets.

9. A candle is 18 centimeters tall after 2 hours and 12 after 5 hours. Find height y at time x.

10. A sensor uses $y = 0.4x + 1.2$. Find the input producing output 3.8, and give the input to the nearest tenth.

Answer Key

1. $y = 2x + 6$.
2. $V = 40 - 3t$; $V(8) = 16$ liters.
3. $y = 3x + 5$.
4. Initial height 7 centimeters; $w = 8$ weeks.
5. $S = 6w + 12$ dollars.
6. A: start $\$8$, rate $\$4$/hour, cost $\$16$; B: start $\$2$, rate $\$6$/hour, cost $\$14$.
7. $y = \frac{9}{5}x + 32$.
8. $B = 7t + 25$; $\$88$.
9. $y = -2x + 22$.
10. $x = 6.5$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The distance rate is $m = 2$ dollars per kilometer. The initial fee is paid at $x = 0$, so the line contains $(0, 6)$.

2. Substitute the known point into $y = mx + b$: $6 = (2)(0) + b$. Since the product is 0, $b = 6$.

3. The equation is $y = 2x + 6$. Check the known point: $(2)(0) + (6) = 6$, so the line has the required point and slope.

Problem 2

1. The initial amount is 40 liters and the signed rate is -3 liters per minute. Therefore the model is $V(t) = 40 - 3t$.

2. At $t = 8$, substitute to obtain $V(8) = 40 - 3(8) = 40 - 24 = 16$ liters. The amount is still nonnegative.

3. The physical interval ends when $40 - 3t = 0$, or $t = \frac{40}{3}$ minutes. Since $8 < \frac{40}{3}$, the prediction is within this model's valid interval.

Problem 3

1. Compute the per-poster rate from the differences: $m = (23 - 11)/(6 - 2) = 12/4 = 3$. The fee may also include a fixed starting charge.

2. Substitute the known point into $y = mx + b$: $11 = (3)(2) + b$. Since the product is 6, $b = 5$.

3. The equation is $y = 3x + 5$. Check the known point: $(3)(2) + (5) = 11$, so the line has the required point and slope.

Problem 4

1. At $w = 0$, $h = 1.5(0) + 7 = 7$, so the constant represents the plant's initial height rather than its weekly growth.

2. For the target height, solve $19 = 1.5w + 7$. Subtract 7 to get $12 = 1.5w$, then divide: $w = 12/1.5 = 8$.

3. Check $1.5(8) + 7 = 12 + 7 = 19$ centimeters. Thus the model reaches the specified height after exactly 8 weeks.

Problem 5

1. The required increase is $42 - 12 = 30$ dollars over 5 weeks. Divide to find weekly deposit $m = 30/5 = 6$ dollars.

2. The starting amount is the intercept $b = 12$. Combining rate and start gives $S(w) = 6w + 12$, with whole nonnegative weeks for deposit counts.

3. Check both endpoints: $S(0) = 12$ and $S(5) = 6(5) + 12 = 42$. The designed model satisfies the two required balances.

Problem 6

1. In $A = 4h + 8$, the rate is 4 and the start is 8. In $B = 6h + 2$, the rate is 6 and the start is 2.

2. At $h = 2$, calculate $A = 4(2) + 8 = 16$ dollars and $B = 6(2) + 2 = 14$ dollars. These include fixed and variable charges.

3. Plan B is cheaper by $16 - 14 = 2$ dollars at two hours, despite its greater hourly rate. A smaller rate alone does not guarantee lower cost.

Problem 7

1. The conversion slope is specified as $\frac{9}{5}$ Fahrenheit degrees per Celsius degree. The known freezing reading provides the point $(0, 32)$.

2. Substitute the known point into $y = mx + b$: $32 = (\frac{9}{5})(0) + b$. Since the product is 0, $b = 32$.

3. The equation is $y = \frac{9}{5}x + 32$. Check the known point: $(\frac{9}{5})(0) + (32) = 32$, so the line has the required point and slope.

Problem 8

1. The starting balance is 25, and each ticket increases it by 7. With ticket count t, the linear model is $B = 7t + 25$.

2. Substitute $t = 9$: $B = 7(9) + 25 = 63 + 25 = 88$. Ticket counts should be nonnegative integers in this application.

3. The ticket revenue is $\$63$ and the original money is $\$25$. Their sum confirms the final balance of $\$88$.

Problem 9

1. The signed burn rate is $m = (12 - 18)/(5 - 2) = -6/3 = -2$ centimeters per hour. Use this rate with one recorded height.

2. Substitute the known point into $y = mx + b$: $18 = (-2)(2) + b$. Since the product is -4, $b = 22$.

3. The equation is $y = -2x + 22$. Check the known point: $(-2)(2) + (22) = 18$, so the line has the required point and slope.

Problem 10

1. Substitute the target output into the model: $3.8 = 0.4x + 1.2$. Subtract the offset 1.2 from each side to isolate the variable term.

2. This gives $2.6 = 0.4x$. Divide by 0.4: $x = 2.6/0.4 = 6.5$, already expressed to the nearest tenth.

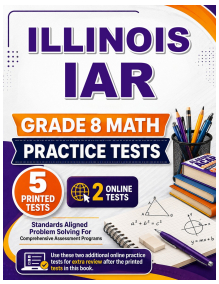
3. Check the input in the original calibration: $0.4(6.5) + 1.2 = 2.6 + 1.2 = 3.8$. Therefore the required input is 6.5.

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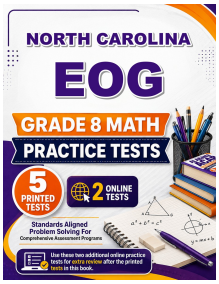
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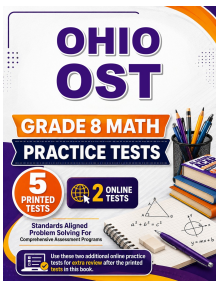
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