Rational and Irrational Numbers

Exact Numbers in Design

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Exact measurements can be rational or irrational. A square with area 10 has side $\sqrt{10}$, while area 16 gives side $4 = \frac{4}{1}$. Circumference $2\pi r$ is irrational for nonzero rational r. Keep exact expressions separate from decimal approximations. To design a length between positive bounds, compare their squares before choosing a radicand.

Worked example

Design a square with integer area between 10 and 20 and rational side.

1. Check nearby squares: $3^2 = 9$, $4^2 = 16$, and $5^2 = 25$. Only 16 lies strictly between 10 and 20.

2. Use area 16 square units. Its positive side is $\sqrt{16} = 4 = \frac{4}{1}$ units, a rational value satisfying the design condition.

Answer: Area 16, side 4.

Practice Problems

Show your thinking.

1. A square tile has area 12 cm^2. Is its exact side length rational?

2. A 3-meter rope is shared equally among 8 groups. Classify each exact share.

3. Design a square with whole-number area between 40 and 60 cm^2 and rational side length.

4. A circle has radius 2 cm. Classify its exact circumference $C = 2\pi r$.

5. A sensor displays 1.414. Does that display prove the true length is $\sqrt{2}$?

6. A mosaic needs a rational length and an irrational length, both between 2 and 3 cm. Propose both.

7. A wheel turns $0.\overline{3}$ revolution each second. Classify the exact turns in 3 seconds.

8. A rectangular panel has diagonal $\sqrt{65}$ cm. Which whole-centimeter marks enclose it?

9. A designer claims a square of area 2 m^2 can have exact side 1.4 m. Test it.

10. Choose an exact rational length from $\sqrt{45}$, $\sqrt{64}$, and $\sqrt{70}$ cm. Explain.

Answer Key

1. No; $\sqrt{12}$ cm is irrational.
2. $\frac{3}{8}$ m $= 0.375$ m; rational.
3. Area 49 cm^2, side 7 cm.
4. 4π cm; irrational.
5. No; $1.414^2 = 1.999396 \neq 2$.
6. $\frac{5}{2}$ cm and $\sqrt{7}$ cm.
7. 1 revolution; rational.
8. Between 8 and 9 cm.
9. No; $1.4^2 = 1.96$ m^2.
10. $\sqrt{64} = 8$ cm.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Let the positive side be s. The area equation $s^2 = 12$ gives $s = \sqrt{12}$ centimeters.

2. The bounds $3^2 = 9 < 12 < 16 = 4^2$ show that 12 is not a perfect square. Therefore the exact side is irrational, although it can be approximated.

Problem 2

1. Equal sharing gives $3 \div 8 = \frac{3}{8}$ meter per group. This fraction directly expresses the measurement as a ratio of integers.

2. Long division gives $\frac{3}{8} = 0.375$. The terminating decimal represents the same exact rational length; it is not a rounded measurement in this model.

Problem 3

1. The neighboring candidate squares are $6^2 = 36$, $7^2 = 49$, and $8^2 = 64$. Only 49 satisfies $40 < 49 < 60$.

2. Choose area 49 cm^2. Its positive side is $\sqrt{49} = 7 = \frac{7}{1}$ centimeters, meeting the rational-side and whole-number-area requirements.

Problem 4

1. Substitute the radius into the circumference formula: $C = 2\pi(2) = 4\pi$ centimeters. This expression is exact.

2. If 4π were rational, dividing it by 4 would make π rational, which is false. Hence the exact circumference is irrational.

Problem 5

1. The displayed finite decimal is exactly $1.414 = \frac{707}{500}$, a rational number if interpreted as an exact value.

2. Squaring gives $1.414^2 = 1.999396$, not 2. The display may approximate $\sqrt{2}$, but the finite reading alone cannot establish that exact irrational length.

Problem 6

1. Choose $\frac{5}{2} = 2.5$, a rational number strictly between 2 and 3. Its denominator is the nonzero integer 2.

2. Choose $\sqrt{7}$ because $2^2 = 4 < 7 < 9 = 3^2$. The integer 7 is not square, so this second length is irrational and also meets the bounds.

Problem 7

1. Let $x = 0.333\ldots$. Subtract x from $10x = 3.333\ldots$ to obtain $9x = 3$, so $x = \frac{1}{3}$.

2. At this constant rate, $3(\frac{1}{3}) = 1$ revolution. The repeating rate and its whole-number total are both rational numbers.

Problem 8

1. The adjacent perfect squares are $8^2 = 64$ and $9^2 = 81$. The radicand satisfies $64 < 65 < 81$.

2. Taking positive square roots gives $8 < \sqrt{65} < 9$. These strict bounds locate the diagonal without replacing its exact irrational length by a rational value.

Problem 9

1. Square the proposed side: $(1.4)^2 = 1.96$. This gives an area 0.04 m^2 less than the required 2 m^2.

2. The correct exact positive side is $\sqrt{2}$ m, which is irrational. The rational length 1.4 m is only an approximation and fails the exact-area requirement.

Problem 10

1. Compute $8^2 = 64$, so the middle candidate is $8 = \frac{8}{1}$, a rational length. It meets the exact rational requirement.

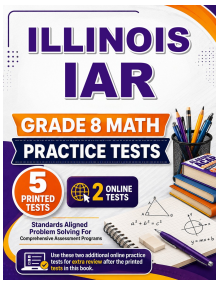
2. The integers 45 and 70 are not perfect squares: $36 < 45 < 49$ and $64 < 70 < 81$. Their square roots are irrational and do not qualify.

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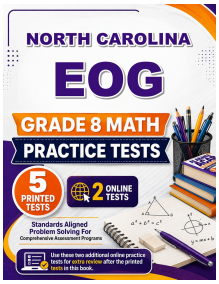
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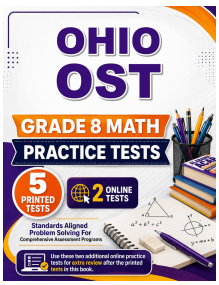
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