Rational and Irrational Numbers

Challenge the Classification

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Test a classification with an exact equality, not a rounded display. Although $\sqrt{3} \approx 1.732$, 1.732 is rational and $\sqrt{3}$ is irrational. Two irrational numbers can have a rational sum or product: $\sqrt{3} - \sqrt{3} = 0$ and $\sqrt{3} \cdot \sqrt{3} = 3$. One counterexample disproves an "always claim. The principal square root is nonnegative.

Worked example

Disprove: "The product of two irrational numbers is always irrational.

1. Since $2^2 < 5 < 3^2$, 5 is not a perfect square and both chosen factors $\sqrt{5}$ are irrational.

2. Nevertheless $(\sqrt{5})^2 = 5 = \frac{5}{1}$ is rational. This verified pair contradicts the word "always, so the original universal claim is false.

Answer: $\sqrt{5} \cdot \sqrt{5} = 5$ is a counterexample.

Practice Problems

Show your thinking.

1. Correct: "Every square root is irrational; for example, $\sqrt{121}$.

2. A student says $0.\overline{6} = 0.6$. Find both fractions to test the claim.

3. Find an integer n with $20 < n < 30$ such that $\sqrt{n}$ is rational.

4. Give an irrational number strictly between 5 and 6, using a square root.

5. Correct: "$\sqrt{2} + \sqrt{2}$ is rational because two irrationals always add to a rational.

6. Can two irrational numbers multiply to 7? Give a verified example.

7. Explain why rounding $\sqrt{11}$ to 3.32 does not make $\sqrt{11}$ rational.

8. Find a rational number strictly between $\frac{1}{3}$ and $\frac{1}{2}$.

9. Which claim is false: $\sqrt{36} = 6$, $\sqrt{36} = -6$, or $(-6)^2 = 36$?

10. Is $\sqrt{50}/\sqrt{2}$ irrational? Simplify before classifying.

Answer Key

1. False: $\sqrt{121} = 11$ is rational.
2. False: $\frac{2}{3} \neq \frac{3}{5}$.
3. $n = 25$.
4. For example, $\sqrt{30}$.
5. False: $2\sqrt{2}$ is irrational.
6. Yes: $\sqrt{7} \cdot \sqrt{7} = 7$.
7. 3.32 is rational; $\sqrt{11}$ remains irrational.
8. For example, $\frac{5}{12}$.
9. $\sqrt{36} = -6$ is false.
10. Rational: 5.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Compute $11^2 = 121$, so the principal square root in the proposed example equals 11, not an irrational decimal.

2. Write $11 = \frac{11}{1}$ to demonstrate rationality. Square roots of nonnegative perfect squares are counterexamples to the claim about every square root.

Problem 2

1. For $x = 0.666\ldots$, $10x = 6.666\ldots$ and $9x = 6$, giving $x = \frac{6}{9} = \frac{2}{3}$.

2. The terminating decimal is $0.6 = \frac{6}{10} = \frac{3}{5}$. Cross-products $2 \cdot 5 = 10$ and $3 \cdot 3 = 9$ differ, so these values are unequal.

Problem 3

1. The consecutive squares around this interval are $4^2 = 16$, $5^2 = 25$, and $6^2 = 36$. Only 25 lies strictly between 20 and 30.

2. Choose $n = 25$. Then $\sqrt{n} = 5 = \frac{5}{1}$, which is rational and satisfies both strict bounds on n.

Problem 4

1. Square the positive endpoints: $5^2 = 25$ and $6^2 = 36$. Choose the nonsquare integer 30 strictly between these squares.

2. Then $25 < 30 < 36$ gives $5 < \sqrt{30} < 6$. Since 30 is not a perfect square, the constructed number is irrational.

Problem 5

1. Combine equal terms: $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$. If this sum were rational, dividing it by 2 would give a rational number.

2. That quotient is $\sqrt{2}$, which is irrational. Thus this particular sum is irrational, and the proposed rule about all irrational sums is false.

Problem 6

1. The integer 7 lies between $2^2 = 4$ and $3^2 = 9$, so each factor $\sqrt{7}$ is irrational.

2. Use the defining square-root identity: $\sqrt{7} \cdot \sqrt{7} = (\sqrt{7})^2 = 7$. The rational product does not require either factor to be rational.

Problem 7

1. The rounded decimal equals $3.32 = \frac{332}{100} = \frac{83}{25}$, so the approximation is rational.

2. But $3.32^2 = 11.0224 \neq 11$. The approximation is not the exact square root; the nonsquare integer 11 still has an irrational square root.

Problem 8

1. Rewrite the bounds with denominator 12: $\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{2} = \frac{6}{12}$.

2. The integer 5 lies strictly between 4 and 6, so $\frac{4}{12} < \frac{5}{12} < \frac{6}{12}$. This constructed ratio is rational and meets both bounds.

Problem 9

1. Both $6^2 = 36$ and $(-6)^2 = 36$ are true computations. They show two solutions of the equation $x^2 = 36$.

2. The symbol $\sqrt{36}$ specifies only the nonnegative root, 6. Thus the negative-root statement is false even though squaring -6 gives 36.

Problem 10

1. The denominator is positive, so combine radicals: $\frac{\sqrt{50}}{\sqrt{2}} = \sqrt{\frac{50}{2}} = \sqrt{25}$.

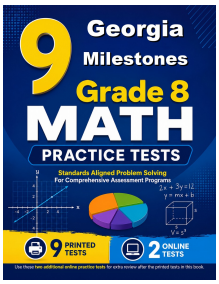
2. Since $5^2 = 25$, the quotient equals $5 = \frac{5}{1}$. Irrational numerator and denominator do not force an irrational quotient.

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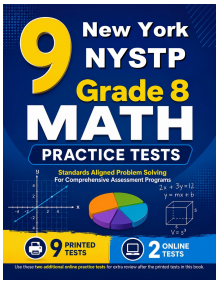
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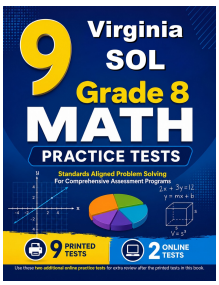
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