Rational and Irrational Numbers

Exact Values and Number Types

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

A rational number is $\frac{a}{b}$ for integers a, b with $b \neq 0$. Its decimal terminates or eventually repeats: $0.125 = \frac{1}{8}$ and $0.\overline{3} = \frac{1}{3}$. An irrational decimal does neither. For a nonnegative integer n, $\sqrt{n}$ is rational exactly when n is a perfect square. Radical notation alone does not determine the classification.

Worked example

Classify $\sqrt{\frac{25}{4}}$.

1. The nonnegative fraction $\frac{5}{2}$ satisfies $(\frac{5}{2})^2 = \frac{25}{4}$, so it is the principal square root.

2. Therefore $\sqrt{\frac{25}{4}} = \frac{5}{2}$. This is a ratio of integers with nonzero denominator, so the radical represents a rational number.

Answer: $\frac{5}{2}$; rational.

Practice Problems

Show your thinking.

1. Classify $\sqrt{81}$ as rational or irrational. Give an exact justification.

2. Classify $-\sqrt{18}$ as rational or irrational.

3. Express 0.375 as a fraction and classify it.

4. Classify $0.\overline{27}$ and give its exact fraction.

5. Classify $\frac{\sqrt{49}}{4}$ and find its exact value.

6. Which is irrational: $\sqrt{0.64}$ or $\sqrt{6}$? Justify both.

7. Classify $\frac{22}{7}$ and π separately. Are they equal?

8. Classify $\sqrt{\frac{9}{16}}$ and give its exact value.

9. Order $\sqrt{5}$, 2.2, and $\frac{9}{4}$ from least to greatest.

10. The decimal $0.101001000100001\ldots$ has successively longer zero runs. Classify it.

Answer Key

1. Rational: 9.
2. Irrational.
3. $\frac{3}{8}$; rational.
4. $\frac{3}{11}$; rational.
5. $\frac{7}{4}$; rational.
6. Only $\sqrt{6}$ is irrational.
7. $\frac{22}{7}$ rational; π irrational; unequal.
8. $\frac{3}{4}$; rational.
9. $2.2 < \sqrt{5} < \frac{9}{4}$.
10. Irrational.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The radicand is a perfect square because $9 \cdot 9 = 81$, so the principal square root is 9.

2. Write the result as $9 = \frac{9}{1}$. This is a ratio of integers with a nonzero denominator, so it is rational.

Problem 2

1. The neighboring integer squares are $4^2 = 16$ and $5^2 = 25$; 18 is not an integer square.

2. Thus $\sqrt{18}$ is irrational. Negating it gives $-\sqrt{18}$, which remains irrational; otherwise its negative would make $\sqrt{18}$ rational.

Problem 3

1. Three decimal places give $0.375 = \frac{375}{1000}$. Both entries are integers, so this already proves rationality.

2. Divide numerator and denominator by 125: $\frac{375}{1000} = \frac{3}{8}$. The simplified exact value is rational, not merely an approximation.

Problem 4

1. Let $x = 0.272727\ldots$. Then $100x = 27.272727\ldots$; subtracting the aligned repeating tails gives $99x = 27$.

2. Dividing yields $x = \frac{27}{99} = \frac{3}{11}$. This exact ratio of integers proves that the repeating decimal is rational.

Problem 5

1. Since $7^2 = 49$, the principal square root is $\sqrt{49} = 7$. Substitute to obtain $\frac{\sqrt{49}}{4} = \frac{7}{4}$.

2. The numerator 7 and denominator 4 are integers, and $4 \neq 0$. Therefore the exact result is rational despite the original radical symbol.

Problem 6

1. Because $0.8^2 = 0.64$, $\sqrt{0.64} = 0.8 = \frac{4}{5}$. This first number is a rational value.

2. For the second, $2^2 = 4 < 6 < 9 = 3^2$. The integer 6 is not a perfect square, so $\sqrt{6}$ is irrational.

Problem 7

1. The expression $\frac{22}{7}$ is a ratio of integers with $7 \neq 0$, so it is rational by definition.

2. The number π is irrational; its decimal neither terminates nor eventually repeats. A rational number cannot equal an irrational number, so $\pi \neq \frac{22}{7}$.

Problem 8

1. Check the nonnegative candidate: $(\frac{3}{4})^2 = \frac{9}{16}$. Therefore the principal square root is exactly $\frac{3}{4}$.

2. The result is a fraction of integers with denominator $4 \neq 0$. Thus this square root is rational even though the radicand is not an integer.

Problem 9

1. All values are positive. Square the rational endpoints: $2.2^2 = 4.84$ and $(\frac{9}{4})^2 = 5.0625$.

2. Since $4.84 < 5 < 5.0625$, taking positive square roots gives $2.2 < \sqrt{5} < 2.25$. Replace 2.25 by its exact fraction $\frac{9}{4}$.

Problem 10

1. The displayed rule gives runs of $1, 2, 3, 4, \ldots$ zeros between successive ones, so the decimal does not terminate.

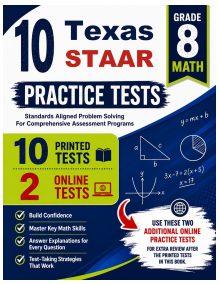
2. Any proposed period of length k fails when a zero run exceeds k, since another 1 still follows. Thus $0.1010010001\ldots$ is not eventually periodic and is irrational.

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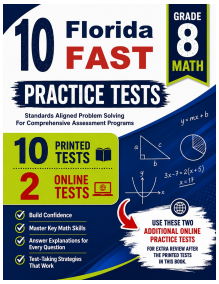
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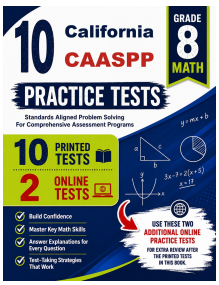
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