Properties of Integer Exponents

Exponents in Designs and Models

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Exponents model repeated factors. An array multiplies its row and column counts; equal packets divide total size by packet count. Square area raises a length to the second power, and cube volume raises it to the third. Repeated halving gives $(\frac{1}{2})^n = 2^{-n}$. Keep units in the conclusion and verify a design by substituting its proposed dimensions or stage count.

Worked example

A cube has edge 3^2 cm. Find its volume as a power and a number.

1. Volume is edge cubed, so $V = (3^2)^3 = 3^{2\cdot 3} = 3^6$. The dimensions contribute six factors of 3 altogether.

2. Evaluate $3^6 = 729$. Alternatively the edge is 9 cm, and $9^3 = 729$ cm^3, confirming the same volume.

Answer: $3^6 = 729$ cm^3.

Practice Problems

Show your thinking.

1. A grid has 2^5 rows and 2^4 columns. Express the number of cells as one power and evaluate.

2. A file has 10^6 bytes and is split into 10^3 equal packets. How many bytes are in each packet?

3. A square has side 3^3 cm. Write its area as a power of 3 and evaluate.

4. A cube has edge 2^2 cm. Write its volume as a power of 2 and evaluate.

5. A filter keeps $\frac{1}{2}$ of a signal at each stage. What fraction remains after 4 stages? Use a negative exponent.

6. Design a rectangular array of 2^{10} cells with 2^3 rows. Specify the number of columns as a power and an integer.

7. A population starts at 3^2 and triples each day. After how many days will it equal 3^6?

8. Two storage blocks hold 2^8 bytes each. A designer claims together they hold 2^{16} bytes. Correct the total.

9. A microscope scale unit is 10^{-3} m. A sample is 10^2 such units long. Find its length as a decimal in meters.

10. A design needs a cube with volume 5^6 cm^3. Give the edge length as a power of 5 and an integer.

Answer Key

1. $2^9 = 512$ cells.
2. $10^3 = 1000$ bytes.
3. $3^6 = 729$ cm^2.
4. $2^6 = 64$ cm^3.
5. $2^{-4} = \frac{1}{16}$.
6. $2^7 = 128$ columns.
7. 4 days.
8. $2^9 = 512$ bytes.
9. 10^{-1} m $= 0.1$ m.
10. $5^2 = 25$ cm.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Multiply row count by column count: $2^5 \cdot 2^4 = 2^{5+4} = 2^9$. Each cell corresponds to one row-column pairing.

2. Evaluate $2^9 = 512$. Checking with the dimensions gives $32 \cdot 16 = 512$, so the grid contains 512 cells.

Problem 2

1. Divide the total size by the number of packets: $\frac{10^6}{10^3} = 10^{6-3} = 10^3$. This uses equal packet sizes.

2. Thus each packet has 1000 bytes. Recombining gives $10^3 \cdot 10^3 = 10^6$ bytes, matching the original file size.

Problem 3

1. Square the side length: $A = (3^3)^2 = 3^{3 \cdot 2} = 3^6$. The exponent doubles because the side is multiplied by itself.

2. Evaluate $3^6 = 729$. The actual side is 27 cm and $27^2 = 729$ cm^2, confirming both the value and the area units.

Problem 4

1. Cube the edge: $V = (2^2)^3 = 2^{2 \cdot 3} = 2^6$. The three dimensions each contribute two factors of 2.

2. Evaluate $2^6 = 64$. Since the edge is 4 cm, the direct check $4 \cdot 4 \cdot 4 = 64$ cm^3 verifies the volume.

Problem 5

1. Each stage multiplies the previous amount by $\frac{1}{2}$. Four stages retain $(\frac{1}{2})^4 = \frac{1}{2^4} = 2^{-4}$ of the initial signal.

2. Evaluate $2^4 = 16$, giving fraction $\frac{1}{16}$. This positive fraction measures the remaining signal; the negative exponent does not mean a negative signal.

Problem 6

1. Columns equal total cells divided by rows: $\frac{2^{10}}{2^3} = 2^{10-3} = 2^7$. This quotient must be a whole-number count.

2. Use 128 columns because $2^7 = 128$. The check $2^3 \cdot 2^7 = 2^{10}$ verifies that the proposed array has exactly the required number of cells.

Problem 7

1. After d days the model is $3^2 \cdot 3^d = 3^{2+d}$. Matching the target 3^6 gives exponent equation $2 + d = 6$.

2. Solve $d = 4$. Checking $9 \cdot 3^4 = 9 \cdot 81 = 729 = 3^6$ confirms that four daily triples reach the stated target.

Problem 8

1. The blocks are added, not multiplied: $2^8 + 2^8 = 2(2^8)$. There are two equal groups of 256 bytes.

2. Write the factor 2 as 2^1: $2^1 \cdot 2^8 = 2^9 = 512$. The proposed 2^{16} would incorrectly multiply the two block sizes.

Problem 9

1. Multiply the count by the unit length: $10^2 \cdot 10^{-3} = 10^{2+(-3)} = 10^{-1}$ meter. The exponents combine because the bases agree.

2. The reciprocal $10^{-1} = \frac{1}{10} = 0.1$. Thus 100 units of 0.001 meter each give a total sample length of 0.1 meter.

Problem 10

1. Write the edge as 5^k. Then its volume is $(5^k)^3 = 5^{3k}$, so matching 5^6 requires $3k = 6$.

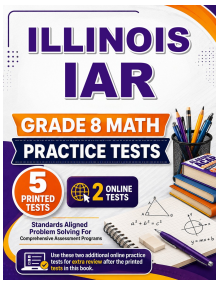
2. Thus $k = 2$, and the edge is $5^2 = 25$ centimeters. Check $25^3 = 15625 = 5^6$ cubic centimeters, satisfying the volume requirement.

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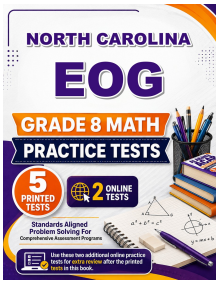
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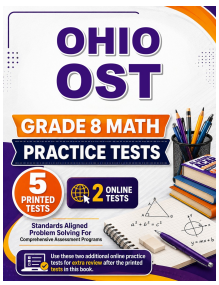
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