Multiplying Binomials and Factoring Quadratics

Binomials: Area, Border, and Product Designs

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Area models turn side-length sums into products: a rectangle with sides $x + 2$ and $x + 5$ has area $x^2 + 7x + 10$. Keep length units separate from square area units. Factoring an area expression can recover possible dimensions, but the dimensions must be positive in context. When a border changes both dimensions, expand the new product before subtracting the original area.

Worked example

A rectangle measures $(x + 2)$ by $(x + 6)$ centimeters. Write its area as a polynomial in square centimeters.

1. Distribute each term in $(x + 2)$ across $(x + 6)$. The first and last products are $(1x)(1x) = 1x^2$ and $(2)(6) = 12$.

2. The cross products are $(1x)(6) = 6x$ and $(2)(1x) = 2x$. Adding these like terms gives $(6 + (2))x = 8x$.

3. Combine the quadratic, linear, and constant terms: $x^2 + 8x + 12$. At $x = 1$, the product is $(3)(7) = 21$, matching $1 + (8) + (12)$.

Answer: $x^2 + 8x + 12$.

Practice Problems

Show your thinking.

1. A garden has side lengths $(x + 1)$ and $(x + 8)$ meters. Expand its area expression; assume $x > 0$.

2. A rectangle has area $x^2 + 11x + 24$ square meters. Factor to give possible side lengths for $x > 0$.

3. A square of side x meters gets a 1-meter border on every side. Find the border area as a polynomial.

4. A rectangle has sides $(x - 2)$ and $(x + 2)$ centimeters, with $x > 2$. Compare its area with a square of side x.

5. Design a rectangle with side lengths $x + a, x + b$, where positive integers a, b give area $x^2 + 12x + 35$.

6. A stage is $(2x + 3)$ by $(x + 2)$ meters. Find its area when $x = 4$ using an expanded polynomial.

7. A panel area is $x^2 - 4x - 21$ square units. Factor to find dimensions and state the restriction for both to be positive.

8. A box base has dimensions $(x + 4)$ and $(x + 1)$ centimeters. A student gives area $x^2 + 4$. Correct and check at $x = 2$.

9. A rectangular floor has area $x^2 + 6x + 8$ square meters and one side $x + 2$ meters. Find the other side for $x > 0$.

10. Design a square whose side is $x + a$ units and area has linear term $14x$. Find positive integer a and its full area polynomial.

Answer Key

1. $x^2 + 9x + 8$.
2. $(x + 3)(x + 8)$.
3. $4x + 4$ square meters.
4. The rectangle has area $x^2 - 4$, exactly 4 square centimeters less.
5. Use $(x + 5)$ and $(x + 7)$.
6. $2x^2 + 7x + 6$; 66 square meters.
7. $(x - 7)(x + 3)$.
8. Area $x^2 + 5x + 4$; at $x = 2$, 18 square centimeters.
9. $x + 4$ meters.
10. $a = 7$; area $x^2 + 14x + 49$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Distribute each term in $(x + 1)$ across $(x + 8)$. The first and last products are $(1x)(1x) = 1x^2$ and $(1)(8) = 8$.

2. The cross products are $(1x)(8) = 8x$ and $(1)(1x) = 1x$. Adding these like terms gives $(8 + (1))x = 9x$.

3. Combine the quadratic, linear, and constant terms: $x^2 + 9x + 8$. At $x = 1$, the product is $(2)(9) = 18$, matching $1 + (9) + (8)$.

Problem 2

1. Choose side-length constants $3, 8$: their sum is 11 and product is 24. For $x > 0$, both proposed dimensions $x + 3$ and $x + 8$ are positive.

2. Check the outer terms: $(1x)(1x) = 1x^2$ and $(3)(8) = 24$. The cross products are $8x$ and $3x$, summing to $11x$.

3. Therefore $x^2 + 11x + 24 = (x + 3)(x + 8)$. Expanding the two factors recovers all three original coefficients, so the factorization is exact.

Problem 3

1. The border adds 1 meter at each end of each dimension, so the outer side is $x + 2$. Its area is $(x + 2)^2$.

2. Expand the outer area:
$(x + 2)^2 = x^2 + 2x + 2x + 4 = x^2 + 4x + 4$. The original inner area is x^2.

3. Subtract inner from outer: $(x^2 + 4x + 4) - x^2 = 4x + 4$ square meters. This counts only the border, including its corner squares.

Problem 4

1. The rectangle's area is
$(x - 2)(x + 2) = x^2 + 2x - 2x - 4 = x^2 - 4$. The condition $x > 2$ keeps both side lengths positive.

2. The comparison square has area x^2. Subtract rectangle area from square area: $x^2 - (x^2 - 4) = 4$ square centimeters.

3. The difference is constant, independent of x. Therefore the rectangle is always 4 square centimeters smaller for the stated positive-dimension domain.

Problem 5

1. Matching the area requires $a + b = 12$ and $ab = 35$. The factor pair $5, 7$ satisfies both: $5 + 7 = 12$ and $5(7) = 35$.

2. Choose dimensions $x + 5$ and $x + 7$. Their product is
$x^2 + 7x + 5x + 35 = x^2 + 12x + 35$.

3. Thus the designed dimensions reproduce the entire area polynomial. For $x > 0$, both lengths are positive, so the algebraic design is also physically valid.

Problem 6

1. Expand the dimensions:
$(2x + 3)(x + 2) = 2x^2 + 4x + 3x + 6 = 2x^2 + 7x + 6$. The middle terms combine to $7x$.

2. At $x = 4$, the resulting area is
$2(4^2) + 7(4) + 6 = 32 + 28 + 6 = 66$ square meters.

3. Check directly with the side lengths: $2(4) + 3 = 11$ and $4 + 2 = 6$ meters. Their product $11(6) = 66$ confirms the area.

Problem 7

1. The pair $-7, 3$ has sum -4 and product -21. Dimensions $x - 7, x + 3$ are both positive when $x > 7$; use that contextual restriction.

2. Check the outer terms: $(1x)(1x) = 1x^2$ and $(-7)(3) = -21$. The cross products are $3x$ and $-7x$, summing to $-4x$.

3. Therefore $x^2 - 4x - 21 = (x - 7)(x + 3)$. Expanding the two factors recovers all three original coefficients, so the factorization is exact.

Problem 8

1. Expand every product:
$(x + 4)(x + 1) = x^2 + x + 4x + 4 = x^2 + 5x + 4$. The student omitted the cross terms totaling $5x$.

2. At $x = 2$, the corrected polynomial gives
$2^2 + 5(2) + 4 = 4 + 10 + 4 = 18$. The student's expression would give only 8.

3. Direct dimensions are $2 + 4 = 6$ and $2 + 1 = 3$ centimeters, with area $6(3) = 18$. This confirms the corrected area model.

Problem 9

1. Factor the area by finding two constants with sum 6 and product 8. The pair $2, 4$ works because $2 + 4 = 6$ and $2(4) = 8$.

2. Thus $x^2 + 6x + 8 = (x + 2)(x + 4)$. Dividing by the known nonzero side $x + 2$ leaves the other side $x + 4$.

3. Both sides are positive for $x > 0$. Multiplying the proposed dimensions gives $x^2 + 4x + 2x + 8 = x^2 + 6x + 8$, confirming the missing length.

Problem 10

1. Expand the general square: $(x + a)^2 = x^2 + 2ax + a^2$. Matching the required linear term means $2a = 14$.

2. Divide by 2 to get $a = 7$. Then $a^2 = 49$, so the area polynomial is $x^2 + 14x + 49$.

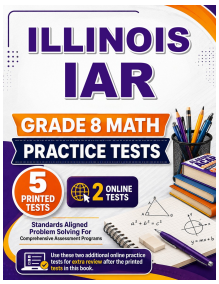
3. Check with the designed side:
$(x + 7)^2 = x^2 + 7x + 7x + 49 = x^2 + 14x + 49$. For $x > 0$, the resulting square has a positive side length.

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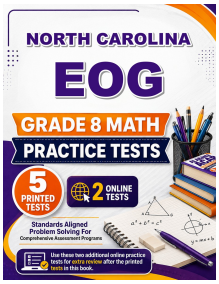
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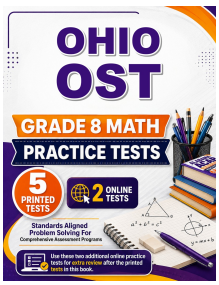
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