Multiplying Binomials and Factoring Quadratics

Binomials: Spot Errors and Reconstruct Factors

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Check both cross products before accepting an expansion. The square $(x+4)^2$ is $x^2+8x+16$, not x^2+16. For monic factoring, a correct constant product alone is insufficient: $(x+1)(x+12)$ has product 12 but middle coefficient 13. Numerical substitution can disprove a claimed identity, while complete expansion verifies every coefficient and establishes equality for all real inputs.

Worked example

A student claims $(x+4)^2 = x^2+16$. Correct the identity.

1. The omitted terms are $x(4) = 4x$ and $4(x) = 4x$. They add to $8x$ rather than disappearing, so the claim lacks a linear term.

2. The cross products are $(1x)(4) = 4x$ and $(4)(1x) = 4x$. Adding these like terms gives $(4+(4))x = 8x$.

3. Combine the quadratic, linear, and constant terms: $x^2+8x+16$. At $x=1$, the product is $(5)(5) = 25$, matching $1+(8)+(16)$.

Answer: $x^2+8x+16$.

Practice Problems

Show your thinking.

1. Correct $(x-3)(x+2) = x^2-6$.

2. A student factors $x^2+8x+15$ as $(x+1)(x+15)$. Repair the factors.

3. Find k so $(x+3)(x+k) = x^2+10x+21$.

4. Fill the missing middle coefficient: $(2x-3)(x+4) = 2x^2+\square x-12$.

5. Is $x^2+9 = (x+3)(x-3)$ an identity? Disprove it numerically and give the correct product.

6. Design a monic quadratic with factors $(x-2)$ and $(x+5)$, and state its coefficients.

7. Factor $x^2-10x+25$. Explain why both signs must be negative.

8. A student stops at $2(x^2+5x+6)$ when factoring $2x^2+10x+12$. Finish completely.

9. Choose two positive integers with product 18 so $(x+a)(x+b)$ has the smallest possible middle coefficient. Give the polynomial.

10. Factor $2x^2+7x+3$ and verify the cross-term sum.

Answer Key

1. $x^2 - x - 6$.
2. $(x + 3)(x + 5)$.
3. $k = 7$.
4. $\square = 5$.
5. No; $(x + 3)(x - 3) = x^2 - 9$.
6. $x^2 + 3x - 10$; coefficients $(1, 3, -10)$.
7. $(x - 5)(x - 5)$.
8. $2(x + 2)(x + 3)$.
9. $a, b = 3, 6$; $x^2 + 9x + 18$.
10. $(2x + 1)(x + 3)$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The first and last products are x^2 and -6, but the cross products $2x$ and $-3x$ are also required. They sum to $-x$.

2. The cross products are $(1x)(2) = 2x$ and $(-3)(1x) = -3x$. Adding these like terms gives $(2 + (-3))x = -1x$.

3. Combine the quadratic, linear, and constant terms: $x^2 - x - 6$. At $x = 1$, the product is $(-2)(3) = -6$, matching $1 + (-1) + (-6)$.

Problem 2

1. The proposed constants multiply to 15 but add to 16, not 8. Choose 3 and 5, since $3(5) = 15$ and $3 + 5 = 8$.

2. Check the outer terms: $(1x)(1x) = 1x^2$ and $(3)(5) = 15$. The cross products are $5x$ and $3x$, summing to $8x$.

3. Therefore $x^2 + 8x + 15 = (x + 3)(x + 5)$. Expanding the two factors recovers all three original coefficients, so the factorization is exact.

Problem 3

1. Expanding the left side gives $x^2 + (3 + k)x + 3k$. Match the linear coefficient to get $3 + k = 10$, so $k = 7$.

2. Check the constant independently: $3k = 3(7) = 21$, exactly the given constant. Both coefficient conditions agree on the same value.

3. Therefore $(x + 3)(x + 7) = x^2 + 7x + 3x + 21 = x^2 + 10x + 21$. The missing number is 7, not merely any factor of 21.

Problem 4

1. The two cross products are $(2x)(4) = 8x$ and $(-3)(x) = -3x$. Their sum determines the missing linear coefficient.

2. Combine $8x - 3x = 5x$. The other products are $(2x)(x) = 2x^2$ and $(-3)(4) = -12$.

3. Thus the complete expansion is $2x^2 + 5x - 12$, so the blank coefficient is 5. All three terms match the displayed structure.

Problem 5

1. Test $x = 0$: the left side is $0^2 + 9 = 9$, while the proposed product is $(0 + 3)(0 - 3) = -9$. The values differ.

2. Expand the product: $x^2 - 3x + 3x - 9 = x^2 - 9$. The cross terms cancel, but the constant product is negative.

3. Therefore the claimed identity is false. Conjugate factors produce a difference of squares $x^2 - 9$, not the sum $x^2 + 9$.

Problem 6

1. Multiply the prescribed factors: $(x - 2)(x + 5) = x^2 + 5x - 2x - 10$. The negative constant comes from $(-2)(5) = -10$.

2. Combine the linear terms $5x - 2x = 3x$, giving $x^2 + 3x - 10$. Its leading coefficient is 1, so it is monic.

3. The coefficients in descending degree order are $1, 3, -10$. Expanding the required factors already verifies that the designed polynomial meets both factor conditions.

Problem 7

1. The constants multiply to positive 25 but sum to negative -10. The pair $-5, -5$ works because $(-5)(-5) = 25$ and $-5 - 5 = -10$.

2. Check the outer terms: $(1x)(1x) = 1x^2$ and $(-5)(-5) = 25$. The cross products are $-5x$ and $-5x$, summing to $-10x$.

3. Therefore $x^2 - 10x + 25 = (x - 5)(x - 5)$. Expanding the two factors recovers all three original coefficients, so the factorization is exact.

Problem 8

1. The extracted common factor is correct: $2x^2 + 10x + 12 = 2(x^2 + 5x + 6)$. The quadratic inside still has integer factors.

2. Choose constants $2, 3$ because $2 + 3 = 5$ and $2(3) = 6$. Thus $x^2 + 5x + 6 = (x + 2)(x + 3)$.

3. The complete result is $2(x + 2)(x + 3)$. Expanding gives $2(x^2 + 5x + 6) = 2x^2 + 10x + 12$, confirming every original coefficient.

Problem 9

1. Positive integer factor pairs of 18 are $(1, 18), (2, 9), (3, 6)$. Their sums are $19, 11, 9$, respectively.

2. The middle coefficient of $(x + a)(x + b)$ is $a + b$. The smallest available sum is $3 + 6 = 9$.

3. Use $(x + 3)(x + 6) = x^2 + 6x + 3x + 18 = x^2 + 9x + 18$. Thus the pair $3, 6$ meets the product condition and minimizes the coefficient.

Problem 10

1. Use factors $(2x + 1)$ and $(x + 3)$: their constant product is $1(3) = 3$, while their cross coefficients are $2(3) = 6$ and $1(1) = 1$.

2. Check the outer terms: $(2x)(1x) = 2x^2$ and $(1)(3) = 3$. The cross products are $6x$ and $1x$, summing to $7x$.

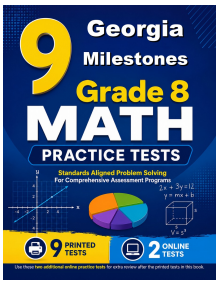
3. Therefore $2x^2 + 7x + 3 = (2x + 1)(x + 3)$. Expanding the two factors recovers all three original coefficients, so the factorization is exact.

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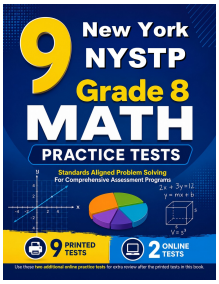
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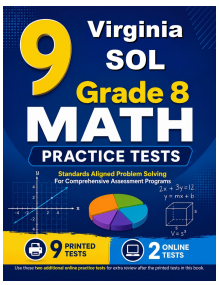
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