Multiplying Binomials and Factoring Quadratics

Binomials: Area Products and Factor Checks

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Distribute every term when multiplying binomials: $(x+2)(x+3)=x^2+3x+2x+6=x^2+5x+6$. The middle coefficient comes from two cross products. Factoring reverses this process; for x^2+bx+c, seek integers whose sum is b and product is c. Special patterns include $(x+a)^2=x^2+2ax+a^2$ and $(x-a)(x+a)=x^2-a^2$. Verify any factors by expansion.

Worked example

Expand $(x+2)(x+5)$ and combine like terms.

1. Distribute each term in $(x+2)$ across $(x+5)$. The first and last products are $(1x)(1x)=1x^2$ and $(2)(5)=10$.

2. The cross products are $(1x)(5)=5x$ and $(2)(1x)=2x$. Adding these like terms gives $(5+(2))x=7x$.

3. Combine the quadratic, linear, and constant terms: $x^2+7x+10$. At $x=1$, the product is $(3)(6)=18$, matching $1+(7)+(10)$.

Answer: $x^2+7x+10$.

Practice Problems

Show your thinking.

1. Expand $(x+3)(x+4)$.

2. Expand $(x-2)(x+7)$.

3. Use four partial products to expand $(2x+1)(x+3)$.

4. Expand $(x-5)^2$. Include both cross products.

5. Expand $(x-4)(x+4)$ and explain the missing linear term.

6. Factor $x^2+7x+10$ completely.

7. Factor x^2-x-12 completely.

8. Factor x^2-36 completely.

9. Factor $3x^2+12x$ completely, then check by multiplication.

10. For $x^2+9x+20=(x+a)(x+b)$, find positive integers a, b and verify.

Answer Key

1. $x^2 + 7x + 12$.
2. $x^2 + 5x - 14$.
3. $2x^2 + 7x + 3$.
4. $x^2 - 10x + 25$.
5. $x^2 - 16$.
6. $(x + 2)(x + 5)$.
7. $(x - 4)(x + 3)$.
8. $(x - 6)(x + 6)$.
9. $3x(x + 4)$.
10. $a, b = 4, 5$ in either order.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Distribute each term in $(x + 3)$ across $(x + 4)$. The first and last products are $(1x)(1x) = 1x^2$ and $(3)(4) = 12$.

2. The cross products are $(1x)(4) = 4x$ and $(3)(1x) = 3x$. Adding these like terms gives $(4 + (3))x = 7x$.

3. Combine the quadratic, linear, and constant terms: $x^2 + 7x + 12$. At $x = 1$, the product is $(4)(5) = 20$, matching $1 + (7) + (12)$.

Problem 2

1. Distribute each term in $(x - 2)$ across $(x + 7)$. The first and last products are $(1x)(1x) = 1x^2$ and $(-2)(7) = -14$.

2. The cross products are $(1x)(7) = 7x$ and $(-2)(1x) = -2x$. Adding these like terms gives $(7 + (-2))x = 5x$.

3. Combine the quadratic, linear, and constant terms: $x^2 + 5x - 14$. At $x = 1$, the product is $(-1)(8) = -8$, matching $1 + (5) + (-14)$.

Problem 3

1. Distribute each term in $(2x + 1)$ across $(x + 3)$. The first and last products are $(2x)(1x) = 2x^2$ and $(1)(3) = 3$.

2. The cross products are $(2x)(3) = 6x$ and $(1)(1x) = 1x$. Adding these like terms gives $(6 + (1))x = 7x$.

3. Combine the quadratic, linear, and constant terms: $2x^2 + 7x + 3$. At $x = 1$, the product is $(3)(4) = 12$, matching $2 + (7) + (3)$.

Problem 4

1. Distribute each term in $(x - 5)$ across $(x - 5)$. The first and last products are $(1x)(1x) = 1x^2$ and $(-5)(-5) = 25$.

2. The cross products are $(1x)(-5) = -5x$ and $(-5)(1x) = -5x$. Adding these like terms gives $(-5 + (-5))x = -10x$.

3. Combine the quadratic, linear, and constant terms: $x^2 - 10x + 25$. At $x = 1$, the product is $(-4)(-4) = 16$, matching $1 + (-10) + (25)$.

Problem 5

1. The opposite constants produce cross terms $4x$ and $-4x$. Their sum is $0x$, so the linear term disappears rather than being omitted accidentally.

2. The cross products are $(1x)(4) = 4x$ and $(-4)(1x) = -4x$. Adding these like terms gives $(4 + (-4))x = 0x$.

3. Combine the quadratic, linear, and constant terms: $x^2 - 16$. At $x = 1$, the product is $(-3)(5) = -15$, matching $1 + (0) + (-16)$.

Problem 6

1. The constant product must be 10 and the linear contributions must sum to $7x$. Choose factors $(x + 2)$ and $(x + 5)$.

2. Check the outer terms: $(1x)(1x) = 1x^2$ and $(2)(5) = 10$. The cross products are $5x$ and $2x$, summing to $7x$.

3. Therefore $x^2 + 7x + 10 = (x + 2)(x + 5)$. Expanding the two factors recovers all three original coefficients, so the factorization is exact.

Problem 7

1. The constant product must be -12 and the linear contributions must sum to $-1x$. Choose factors $(x - 4)$ and $(x + 3)$.

2. Check the outer terms: $(1x)(1x) = 1x^2$ and $(-4)(3) = -12$. The cross products are $3x$ and $-4x$, summing to $-1x$.

3. Therefore $x^2 - x - 12 = (x - 4)(x + 3)$. Expanding the two factors recovers all three original coefficients, so the factorization is exact.

Problem 8

1. Write $36 = 6^2$, so $x^2 - 36 = x^2 - 6^2$. The difference-of-squares pattern suggests the conjugate pair $(x - 6)(x + 6)$.

2. Check the outer terms: $(1x)(1x) = 1x^2$ and $(-6)(6) = -36$. The cross products are $6x$ and $-6x$, summing to $0x$.

3. Therefore $x^2 - 36 = (x - 6)(x + 6)$. Expanding the two factors recovers all three original coefficients, so the factorization is exact.

Problem 9

1. Both terms contain $3x$: $3x^2/(3x) = x$ and $12x/(3x) = 4$. The greatest common monomial factor is $3x$.

2. Take that factor outside parentheses: $3x^2 + 12x = 3x(x + 4)$. No nonconstant factor remains common to both terms inside.

3. Check by distribution: $3x(x) + 3x(4) = 3x^2 + 12x$. The product reproduces the original expression exactly, so the complete factorization is $3x(x + 4)$.

Problem 10

1. The constants must multiply to 20 and add to 9. Test the factor pair $4, 5$: $4(5) = 20$ and $4 + 5 = 9$.

2. Thus $(x + 4)(x + 5) = x^2 + 5x + 4x + 20 = x^2 + 9x + 20$. Both the constant and middle coefficient match.

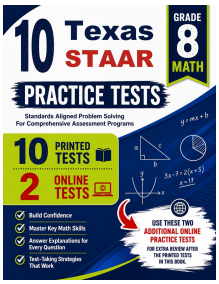
3. Therefore $a = 4, b = 5$, or the reverse order, works. Multiplication is commutative, so switching the factors does not change the polynomial.

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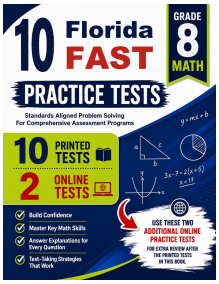
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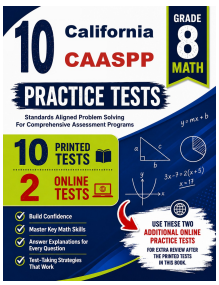
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