Estimating Irrational Numbers

Measure and Design with Root Bounds

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

A square of area A has positive side $\sqrt{A}$, with length units rather than square units. A proposed side s is long enough exactly when $s^2 \geq A$. For a design constraint $a < \sqrt{A} < b$, square the positive bounds to obtain $a^2 < A < b^2$. Round only when requested; exact bounds can decide a fit without rounding.

Worked example

A square of area 55 m^2 needs a side rail. Is 7.5 m sufficient?

1. Square the lower endpoint: $(7)^2 = 49 < 55$. Thus 7 is below the positive root.

2. Square the upper endpoint: $(7.5)^2 = 56.25 > 55$. Thus 7.5 is above the positive root.

3. Squaring increases on nonnegative inputs, so $7 < \sqrt{55} < 7.5$. These endpoints bound, rather than equal, the exact root.

Answer: Yes; $7 < \sqrt{55} < 7.5$ m.

Practice Problems

Show your thinking.

1. A square garden has area 45 m^2. Estimate its side in meters to the nearest tenth.

2. A square screen has area 90 dm^2. Estimate its side in decimeters to the nearest hundredth.

3. A square panel has area 38 m^2. Will its side fit along a 6.2 m opening?

4. A square mat has area 68 m^2. A supplier offers sides 8.0, 8.3, or 8.6 m. Choose the smallest that covers its side.

5. Design a square with whole-number area and side strictly between 3.1 and 3.3 m.

6. A square stage has area 117 m^2. Estimate its side in meters to the nearest integer.

7. A square poster has area 20 dm^2. Name the consecutive tenth marks enclosing its side on a ruler.

8. A computer labels the exact side of an 80 cm^2 square as 8.95 cm. Is that exact? Give hundredth bounds.

9. A square platform must have whole-number area and side strictly between 7.0 and 7.2 m. List all possible areas.

10. A square mural has area 200 ft^2. Estimate its side in feet to the nearest tenth.

Answer Key

1. 6.7
2. 9.49
3. Yes; $6 < \sqrt{38} < 6.2$ m.
4. 8.3 m.
5. Area 10 m^2; side $\sqrt{10}$ m.
6. 11
7. $4.4 < \sqrt{20} < 4.5$
8. Not exact; $8.94 < \sqrt{80} < 8.95$ cm.
9. 50 or 51 m^2.
10. 14.1

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Square the lower endpoint: $(6.65)^2 = 44.2225 < 45$. Thus 6.65 is below the positive root.

2. Square the upper endpoint: $(6.75)^2 = 45.5625 > 45$. Thus 6.75 is above the positive root.

3. Squaring increases on nonnegative inputs, so $6.65 < \sqrt{45} < 6.75$. These endpoints bound, rather than equal, the exact root.

4. Both endpoints are rounding midpoints around 6.7. Since the root lies strictly between them, $\sqrt{45} \approx 6.7$ at the requested place.

Problem 2

1. Square the lower endpoint: $(9.485)^2 = 89.965225 < 90$. Thus 9.485 is below the positive root.

2. Square the upper endpoint: $(9.495)^2 = 90.155025 > 90$. Thus 9.495 is above the positive root.

3. Squaring increases on nonnegative inputs, so $9.485 < \sqrt{90} < 9.495$. These endpoints bound, rather than equal, the exact root.

4. Both endpoints are rounding midpoints around 9.49. Since the root lies strictly between them, $\sqrt{90} \approx 9.49$ at the requested place.

Problem 3

1. Square the lower endpoint: $(6)^2 = 36 < 38$. Thus 6 is below the positive root.

2. Square the upper endpoint: $(6.2)^2 = 38.44 > 38$. Thus 6.2 is above the positive root.

3. Squaring increases on nonnegative inputs, so $6 < \sqrt{38} < 6.2$. These endpoints bound, rather than equal, the exact root.

4. The panel side is $\sqrt{38}$ meters, strictly less than the opening length 6.2 meters. Therefore the side-length requirement is satisfied.

Problem 4

1. Square the lower endpoint: $(8)^2 = 64 < 68$. Thus 8 is below the positive root.

2. Square the upper endpoint: $(8.3)^2 = 68.89 > 68$. Thus 8.3 is above the positive root.

3. Squaring increases on nonnegative inputs, so $8 < \sqrt{68} < 8.3$. These endpoints bound, rather than equal, the exact root.

4. The exact side exceeds 8.0 meters but is less than 8.3 meters. Thus 8.0 is too short and 8.3 is the smallest offered sufficient length.

Problem 5

1. Squaring the desired bounds gives $3.1^2 = 9.61$ and $3.3^2 = 10.89$. A suitable whole-number area must satisfy $9.61 < A < 10.89$.

2. Choose $A = 10$. Taking positive square roots yields $3.1 < \sqrt{10} < 3.3$, so this area produces a side satisfying the complete design requirement.

Problem 6

1. Square the lower endpoint: $(10.5)^2 = 110.25 < 117$. Thus 10.5 is below the positive root.

2. Square the upper endpoint: $(11.5)^2 = 132.25 > 117$. Thus 11.5 is above the positive root.

3. Squaring increases on nonnegative inputs, so $10.5 < \sqrt{117} < 11.5$. These endpoints bound, rather than equal, the exact root.

4. Both endpoints are rounding midpoints around 11. Since the root lies strictly between them, $\sqrt{117} \approx 11$ at the requested place.

Problem 7

1. Square the lower endpoint: $(4.4)^2 = 19.36 < 20$. Thus 4.4 is below the positive root.

2. Square the upper endpoint: $(4.5)^2 = 20.25 > 20$. Thus 4.5 is above the positive root.

3. Squaring increases on nonnegative inputs, so $4.4 < \sqrt{20} < 4.5$. These endpoints bound, rather than equal, the exact root.

Problem 8

1. Square the lower endpoint: $(8.94)^2 = 79.9236 < 80$. Thus 8.94 is below the positive root.

2. Square the upper endpoint: $(8.95)^2 = 80.1025 > 80$. Thus 8.95 is above the positive root.

3. Squaring increases on nonnegative inputs, so $8.94 < \sqrt{80} < 8.95$. These endpoints bound, rather than equal, the exact root.

Problem 9

1. Square the positive side limits: $7.0^2 = 49$ and $7.2^2 = 51.84$. The permitted area satisfies $49 < A < 51.84$.

2. The only whole-number areas are 50 and 51 square meters. Both are nonsquares, and taking their square roots places both sides inside the required interval.

Problem 10

1. Square the lower endpoint: $(14.05)^2 = 197.4025 < 200$. Thus 14.05 is below the positive root.

2. Square the upper endpoint: $(14.15)^2 = 200.2225 > 200$. Thus 14.15 is above the positive root.

3. Squaring increases on nonnegative inputs, so $14.05 < \sqrt{200} < 14.15$. These endpoints bound, rather than equal, the exact root.

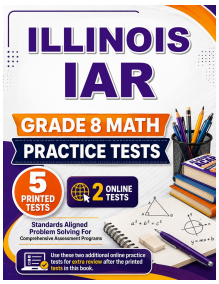
4. Both endpoints are rounding midpoints around 14.1. Since the root lies strictly between them, $\sqrt{200} \approx 14.1$ at the requested place.

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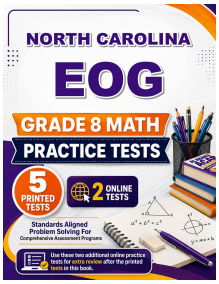
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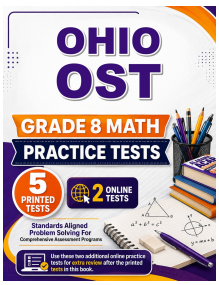
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