Estimating Irrational Numbers

Audit Root Estimates

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QUICK REVIEW

An estimate needs evidence from squares, not just a nearby integer. If $3.4^2 < n < 3.5^2$, then $3.4 < \sqrt{n} < 3.5$; midpoint 3.45 decides nearest-tenth rounding. Reverse questions square both positive bounds to restrict n. Preserve strict endpoints: $4 < \sqrt{n} < 5$ means $16 < n < 25$, so integer candidates exclude 16 and 25.

Worked example

Correct a claim that $\sqrt{29} = 5.2$ using consecutive tenths.

1. Square the lower endpoint: $(5.3)^2 = 28.09 < 29$. Thus 5.3 is below the positive root.

2. Square the upper endpoint: $(5.4)^2 = 29.16 > 29$. Thus 5.4 is above the positive root.

3. Squaring increases on nonnegative inputs, so $5.3 < \sqrt{29} < 5.4$. These endpoints bound, rather than equal, the exact root.

Answer: $5.3 < \sqrt{29} < 5.4$.

Practice Problems

Show your thinking.

1. Correct the claim $\sqrt{15} = 3.5$ using consecutive-tenth bounds.

2. Find every integer n satisfying $6 < \sqrt{n} < 7$.

3. A student rounds $\sqrt{26}$ to 5.0. Correct the nearest-tenth result with midpoint bounds.

4. A student reasons $\sqrt{50} > 7.1$ because $50 > 49$. Test 7.1 directly.

5. Find every integer n with $2.2 < \sqrt{n} < 2.4$.

6. Decide whether $\sqrt{33}$ rounds to 5.7 or 5.8 to the nearest tenth. Test 5.75.

7. Is the exact value $\sqrt{99}$ equal to 9.95? Use squares to decide.

8. An integer radicand n has $\sqrt{n}$ rounding to 4 to the nearest integer. List all possible n.

9. Explain why $1.41^2 < 2$ proves $1.41 < \sqrt{2}$, not $1.41 > \sqrt{2}$. Give an upper bound too.

10. Test the proposed hundredth estimate 7.93 for $\sqrt{63}$ and give the correct one.

Answer Key

1. False; $3.8 < \sqrt{15} < 3.9$.
2. $n = 37, 38, \ldots, 48$.
3. 5.1
4. False; $7 < \sqrt{50} < 7.1$.
5. Only $n = 5$.
6. 5.7.
7. No; $9.94 < \sqrt{99} < 9.95$.
8. $n = 13, 14, \ldots, 20$.
9. $1.41 < \sqrt{2} < 1.42$
10. 7.94

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Square the lower endpoint: $(3.8)^2 = 14.44 < 15$. Thus 3.8 is below the positive root.

2. Square the upper endpoint: $(3.9)^2 = 15.21 > 15$. Thus 3.9 is above the positive root.

3. Squaring increases on nonnegative inputs, so $3.8 < \sqrt{15} < 3.9$. These endpoints bound, rather than equal, the exact root.

4. The proposed 3.5 is below the entire certified interval. In fact $3.5^2 = 12.25 \neq 15$, so it is neither exact nor a correct nearest-tenth estimate.

Problem 2

1. All quantities are positive, so squaring preserves the inequalities: $6^2 < n < 7^2$, which simplifies to $36 < n < 49$.

2. The integers strictly inside that interval are 37 through 48. Neither endpoint qualifies, and none of the included integers is a perfect square.

Problem 3

1. Square the lower endpoint: $(5.05)^2 = 25.5025 < 26$. Thus 5.05 is below the positive root.

2. Square the upper endpoint: $(5.15)^2 = 26.5225 > 26$. Thus 5.15 is above the positive root.

3. Squaring increases on nonnegative inputs, so $5.05 < \sqrt{26} < 5.15$. These endpoints bound, rather than equal, the exact root.

4. Both endpoints are rounding midpoints around 5.1. Since the root lies strictly between them, $\sqrt{26} \approx 5.1$ at the requested place.

Problem 4

1. Square the lower endpoint: $(7)^2 = 49 < 50$. Thus 7 is below the positive root.

2. Square the upper endpoint: $(7.1)^2 = 50.41 > 50$. Thus 7.1 is above the positive root.

3. Squaring increases on nonnegative inputs, so $7 < \sqrt{50} < 7.1$. These endpoints bound, rather than equal, the exact root.

Problem 5

1. Square the positive bounds: $2.2^2 = 4.84$ and $2.4^2 = 5.76$. The required radicand satisfies $4.84 < n < 5.76$.

2. Only the integer 5 lies in this interval. Checking $4.84 < 5 < 5.76$ confirms that $\sqrt{5}$ satisfies both strict original bounds.

Problem 6

1. Square the lower endpoint: $(5.7)^2 = 32.49 < 33$. Thus 5.7 is below the positive root.

2. Square the upper endpoint: $(5.75)^2 = 33.0625 > 33$. Thus 5.75 is above the positive root.

3. Squaring increases on nonnegative inputs, so $5.7 < \sqrt{33} < 5.75$. These endpoints bound, rather than equal, the exact root.

4. The root is below 5.75, the midpoint between 5.7 and 5.8, and above 5.7. Therefore its nearest-tenth value is 5.7.

Problem 7

1. Square the lower endpoint: $(9.94)^2 = 98.8036 < 99$. Thus 9.94 is below the positive root.

2. Square the upper endpoint: $(9.95)^2 = 99.0025 > 99$. Thus 9.95 is above the positive root.

3. Squaring increases on nonnegative inputs, so $9.94 < \sqrt{99} < 9.95$. These endpoints bound, rather than equal, the exact root.

Problem 8

1. Rounding to 4 requires $3.5 \leq \sqrt{n} < 4.5$. Square the nonnegative endpoints to get $12.25 \leq n < 20.25$.

2. The integer radicands are 13 through 20. This list includes the perfect square 16 because the question does not require an irrational root.

Problem 9

1. Square the lower endpoint: $(1.41)^2 = 1.9881 < 2$. Thus 1.41 is below the positive root.

2. Square the upper endpoint: $(1.42)^2 = 2.0164 > 2$. Thus 1.42 is above the positive root.

3. Squaring increases on nonnegative inputs, so $1.41 < \sqrt{2} < 1.42$. These endpoints bound, rather than equal, the exact root.

Problem 10

1. Square the lower endpoint: $(7.935)^2 = 62.964225 < 63$. Thus 7.935 is below the positive root.

2. Square the upper endpoint: $(7.945)^2 = 63.123025 > 63$. Thus 7.945 is above the positive root.

3. Squaring increases on nonnegative inputs, so $7.935 < \sqrt{63} < 7.945$. These endpoints bound, rather than equal, the exact root.

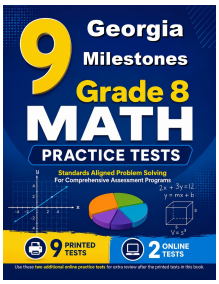
4. Both endpoints are rounding midpoints around 7.94. Since the root lies strictly between them, $\sqrt{63} \approx 7.94$ at the requested place.

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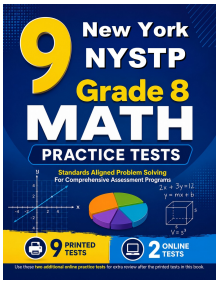
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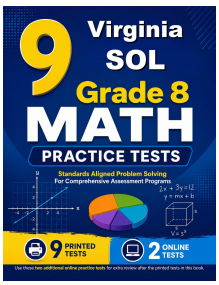
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