Estimating Irrational Numbers

Locate and Round Irrational Roots

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

For $n > 0$, compare squares to locate $\sqrt{n}$: $a^2 < n < b^2$ implies $a < \sqrt{n} < b$ when $0 \le a < b$. For example, $2.8^2 = 7.84 < 8 < 8.41 = 2.9^2$. To round to the nearest tenth, test the midpoint between neighboring tenths. Keep strict bounds distinct from a rounded estimate and write $\approx$ rather than $=$ for approximations.

Worked example

Estimate $\sqrt{17}$ to the nearest tenth.

1. Square the lower endpoint: $(4.05)^2 = 16.4025 < 17$. Thus 4.05 is below the positive root.

2. Square the upper endpoint: $(4.15)^2 = 17.2225 > 17$. Thus 4.15 is above the positive root.

3. Squaring increases on nonnegative inputs, so $4.05 < \sqrt{17} < 4.15$. These endpoints bound, rather than equal, the exact root.

4. Both endpoints are rounding midpoints around 4.1. Since the root lies strictly between them, $\sqrt{17} \approx 4.1$ at the requested place.

Answer: 4.1

Practice Problems

Show your thinking.

1. Place $\sqrt{19}$ between consecutive integers.

2. Which two consecutive whole numbers enclose $\sqrt{58}$?

3. Estimate $\sqrt{13}$ to the nearest tenth.

4. Estimate $\sqrt{31}$ to the nearest hundredth.

5. Prove that $2.64 < \sqrt{7} < 2.65$ by squaring both endpoints.

6. Locate $\sqrt{40}$ between consecutive tenths.

7. Is $\sqrt{10}$ greater than 3.16? Verify a hundredth-wide interval.

8. Is $\sqrt{72}$ closer to 8 or 9? Test the midpoint 8.5.

9. Estimate $\sqrt{85}$ to the nearest integer.

10. Order 4.79, $\sqrt{23}$, and 4.80 without rounding the root.

Answer Key

1. $4 < \sqrt{19} < 5$
2. $7 < \sqrt{58} < 8$
3. 3.6
4. 5.57
5. $2.64 < \sqrt{7} < 2.65$
6. $6.3 < \sqrt{40} < 6.4$
7. Yes; $3.16 < \sqrt{10} < 3.17$.
8. Closer to 8.
9. 9
10. $4.79 < \sqrt{23} < 4.8$

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Square the lower endpoint: $(4)^2 = 16 < 19$. Thus 4 is below the positive root.

2. Square the upper endpoint: $(5)^2 = 25 > 19$. Thus 5 is above the positive root.

3. Squaring increases on nonnegative inputs, so $4 < \sqrt{19} < 5$. These endpoints bound, rather than equal, the exact root.

Problem 2

1. Square the lower endpoint: $(7)^2 = 49 < 58$. Thus 7 is below the positive root.

2. Square the upper endpoint: $(8)^2 = 64 > 58$. Thus 8 is above the positive root.

3. Squaring increases on nonnegative inputs, so $7 < \sqrt{58} < 8$. These endpoints bound, rather than equal, the exact root.

Problem 3

1. Square the lower endpoint: $(3.55)^2 = 12.6025 < 13$. Thus 3.55 is below the positive root.

2. Square the upper endpoint: $(3.65)^2 = 13.3225 > 13$. Thus 3.65 is above the positive root.

3. Squaring increases on nonnegative inputs, so $3.55 < \sqrt{13} < 3.65$. These endpoints bound, rather than equal, the exact root.

4. Both endpoints are rounding midpoints around 3.6. Since the root lies strictly between them, $\sqrt{13} \approx 3.6$ at the requested place.

Problem 4

1. Square the lower endpoint: $(5.565)^2 = 30.969225 < 31$. Thus 5.565 is below the positive root.

2. Square the upper endpoint: $(5.575)^2 = 31.080625 > 31$. Thus 5.575 is above the positive root.

3. Squaring increases on nonnegative inputs, so $5.565 < \sqrt{31} < 5.575$. These endpoints bound, rather than equal, the exact root.

4. Both endpoints are rounding midpoints around 5.57. Since the root lies strictly between them, $\sqrt{31} \approx 5.57$ at the requested place.

Problem 5

1. Square the lower endpoint: $(2.64)^2 = 6.9696 < 7$. Thus 2.64 is below the positive root.

2. Square the upper endpoint: $(2.65)^2 = 7.0225 > 7$. Thus 2.65 is above the positive root.

3. Squaring increases on nonnegative inputs, so $2.64 < \sqrt{7} < 2.65$. These endpoints bound, rather than equal, the exact root.

Problem 6

1. Square the lower endpoint: $(6.3)^2 = 39.69 < 40$. Thus 6.3 is below the positive root.

2. Square the upper endpoint: $(6.4)^2 = 40.96 > 40$. Thus 6.4 is above the positive root.

3. Squaring increases on nonnegative inputs, so $6.3 < \sqrt{40} < 6.4$. These endpoints bound, rather than equal, the exact root.

Problem 7

1. Square the lower endpoint: $(3.16)^2 = 9.9856 < 10$. Thus 3.16 is below the positive root.

2. Square the upper endpoint: $(3.17)^2 = 10.0489 > 10$. Thus 3.17 is above the positive root.

3. Squaring increases on nonnegative inputs, so $3.16 < \sqrt{10} < 3.17$. These endpoints bound, rather than equal, the exact root.

Problem 8

1. Square the lower endpoint: $(8)^2 = 64 < 72$. Thus 8 is below the positive root.

2. Square the upper endpoint: $(8.5)^2 = 72.25 > 72$. Thus 8.5 is above the positive root.

3. Squaring increases on nonnegative inputs, so $8 < \sqrt{72} < 8.5$. These endpoints bound, rather than equal, the exact root.

4. Since $8 < \sqrt{72} < 8.5$, the root is on the 8 side of the midpoint between 8 and 9. Thus it is closer to 8.

Problem 9

1. Square the lower endpoint: $(8.5)^2 = 72.25 < 85$. Thus 8.5 is below the positive root.

2. Square the upper endpoint: $(9.5)^2 = 90.25 > 85$. Thus 9.5 is above the positive root.

3. Squaring increases on nonnegative inputs, so $8.5 < \sqrt{85} < 9.5$. These endpoints bound, rather than equal, the exact root.

4. Both endpoints are rounding midpoints around 9. Since the root lies strictly between them, $\sqrt{85} \approx 9$ at the requested place.

Problem 10

1. Square the lower endpoint: $(4.79)^2 = 22.9441 < 23$. Thus 4.79 is below the positive root.

2. Square the upper endpoint: $(4.8)^2 = 23.04 > 23$. Thus 4.8 is above the positive root.

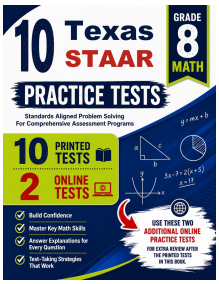
3. Squaring increases on nonnegative inputs, so $4.79 < \sqrt{23} < 4.8$. These endpoints bound, rather than equal, the exact root.

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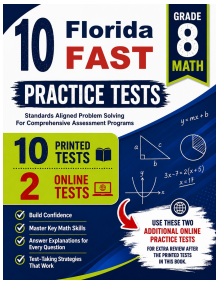
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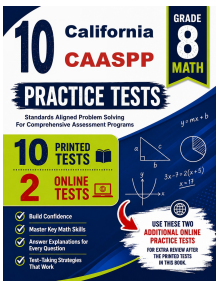
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