Estimating Expressions with Irrational Numbers

Build with Irrational Lengths

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Translate the design into an expression with units: a square of area A has perimeter $4\sqrt{A}$, while a leftover length might be $L - \sqrt{A}$. Propagate lower and upper bounds through the whole expression. A fit decision can use an exact inequality rather than a rounded estimate. For currency round the final total to cents, not each intermediate root.

Worked example

Find the perimeter of a square of area 22 m^2 to the nearest tenth meter.

1. Square the trial bounds: $4.6904^2 = 21.99985216 < 22 < 22.00079025 = 4.6905^2$. Thus $4.6904 < \sqrt{22} < 4.6905$.

2. Substitute both bounds into $4\sqrt{22} + 0$. The endpoint values are $4(4.6904) + (0) = 18.7616$ and $4(4.6905) + (0) = 18.762$.

3. Ordering these results gives $18.7616 < 4\sqrt{22} + 0 < 18.762$. Both bounds round to 18.8, so the requested estimate is 18.8.

Answer: 18.8

Practice Problems

Show your thinking.

1. A square garden has area 30 m^2. Estimate perimeter $4\sqrt{30}$ in meters to the nearest tenth.

2. A path has two sections of $\sqrt{18}$ m and one of 3 m. Estimate total length to the nearest hundredth meter.

3. A 9 m rail is cut by $\sqrt{20}$ m. Estimate the remaining length to the nearest tenth meter.

4. A model fee is $C = 2 + 1.5\sqrt{12}$ dollars. Estimate the fee to the nearest cent.

5. A frame for a square of area 14 m^2 needs perimeter $4\sqrt{14}$ m. Is 15 m of trim enough? Use $3.741 < \sqrt{14} < 3.742$.

6. Design an integer-area square whose perimeter is strictly between 20 and 24 m. Give one nonsquare area.

7. A square plate has area 45 cm^2. Estimate half its side to the nearest hundredth centimeter.

8. A route is $\sqrt{13} + \sqrt{17}$ km. Use $3.605 < \sqrt{13} < 3.606$ and $4.123 < \sqrt{17} < 4.124$ to decide if it is under 7.73 km.

9. A bracket needs length $2\sqrt{50} - 1$ cm. Estimate the cut to the nearest tenth centimeter.

10. A display stand has height $h = k + \sqrt{5}$ m, with whole-number k. Design one with $5 < h < 6$, using $2.23 < \sqrt{5} < 2.24$.

Answer Key

1. 21.9
2. 11.49
3. 4.5
4. 7.20
5. Yes; $14.964 < P < 14.968$ m.
6. For example, $A = 30$ m^2.
7. 3.35
8. Yes; $7.728 < L < 7.730$ km.
9. 13.1
10. $k = 3$; $5.23 < h < 5.24$ m.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Square the trial bounds: $5.4772^2 = 29.99971984 < 30 < 30.00081529 = 5.4773^2$. Thus $5.4772 < \sqrt{30} < 5.4773$.

2. Substitute both bounds into $4\sqrt{30} + 0$. The endpoint values are $4(5.4772) + (0) = 21.9088$ and $4(5.4773) + (0) = 21.9092$.

3. Ordering these results gives $21.9088 < 4\sqrt{30} + 0 < 21.9092$. Both bounds round to 21.9, so the requested estimate is 21.9.

Problem 2

1. Square the trial bounds: $4.2426^2 = 17.99965476 < 18 < 18.00050329 = 4.2427^2$. Thus $4.2426 < \sqrt{18} < 4.2427$.

2. Substitute both bounds into $2\sqrt{18} + 3$. The endpoint values are $2(4.2426) + (3) = 11.4852$ and $2(4.2427) + (3) = 11.4854$.

3. Ordering these results gives $11.4852 < 2\sqrt{18} + 3 < 11.4854$. Both bounds round to 11.49, so the requested estimate is 11.49.

Problem 3

1. Square the trial bounds: $4.4721^2 = 19.99967841 < 20 < 20.00057284 = 4.4722^2$. Thus $4.4721 < \sqrt{20} < 4.4722$.

2. Substitute both bounds into $-1\sqrt{20} + 9$. The endpoint values are $-1(4.4721) + (9) = 4.5279$ and $-1(4.4722) + (9) = 4.5278$.

3. Ordering these results gives $4.5278 < -1\sqrt{20} + 9 < 4.5279$. Both bounds round to 4.5, so the requested estimate is 4.5.

Problem 4

1. Square the trial bounds: $3.4641^2 = 11.99998881 < 12 < 12.00068164 = 3.4642^2$. Thus $3.4641 < \sqrt{12} < 3.4642$.

2. Substitute both bounds into $\frac{3}{2}\sqrt{12} + 2$. The endpoint values are $\frac{3}{2}(3.4641) + (2) = 7.19615$ and $\frac{3}{2}(3.4642) + (2) = 7.1963$.

3. Ordering these results gives $7.19615 < \frac{3}{2}\sqrt{12} + 2 < 7.1963$. Both bounds round to 7.20, so the requested estimate is 7.20.

Problem 5

1. Multiply both root bounds by 4: $4(3.741) = 14.964$ and $4(3.742) = 14.968$. These bound the exact perimeter in meters.

2. Because $P < 14.968 < 15$, the available 15 meters exceeds the exact requirement. Therefore it is enough; no rounded perimeter is needed for this decision.

Problem 6

1. Use $P = 4\sqrt{A}$: dividing $20 < 4\sqrt{A} < 24$ by 4 gives $5 < \sqrt{A} < 6$, hence $25 < A < 36$.

2. Choose the nonsquare integer $A = 30$. Since $25 < 30 < 36$, its perimeter $4\sqrt{30}$ satisfies the design bounds and its side is irrational.

Problem 7

1. Square the trial bounds: $6.7082^2 = 44.99994724 < 45 < 45.00128889 = 6.7083^2$. Thus $6.7082 < \sqrt{45} < 6.7083$.

2. Substitute both bounds into $\frac{1}{2}\sqrt{45} + 0$. The endpoint values are $\frac{1}{2}(6.7082) + (0) = 3.3541$ and $\frac{1}{2}(6.7083) + (0) = 3.35415$.

3. Ordering these results gives $3.3541 < \frac{1}{2}\sqrt{45} + 0 < 3.35415$. Both bounds round to 3.35, so the requested estimate is 3.35.

Problem 8

1. Add lower endpoints: $3.605 + 4.123 = 7.728$. Add upper endpoints: $3.606 + 4.124 = 7.730$. These are strict bounds on the route length.

2. Therefore $7.728 < L < 7.730 = 7.73$. The upper inequality is strict, so the route is under the limit even though rounding could display 7.73.

Problem 9

1. Square the trial bounds: $7.071^2 = 49.999041 < 50 < 50.00045521 = 7.0711^2$. Thus $7.071 < \sqrt{50} < 7.0711$.

2. Substitute both bounds into $2\sqrt{50} - 1$. The endpoint values are $2(7.071) + (-1) = 13.142$ and $2(7.0711) + (-1) = 13.1422$.

3. Ordering these results gives $13.142 < 2\sqrt{50} - 1 < 13.1422$. Both bounds round to 13.1, so the requested estimate is 13.1.

Problem 10

1. The root is between 2.23 and 2.24. Adding 3 gives $3 + 2.23 = 5.23$ and $3 + 2.24 = 5.24$.

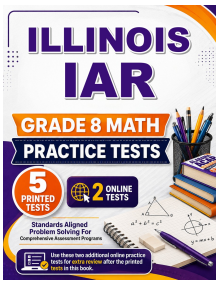
2. Thus choose $k = 3$, yielding $5.23 < h < 5.24$, wholly inside $(5, 6)$. The next smaller or larger integer shift would place the height outside the target interval.

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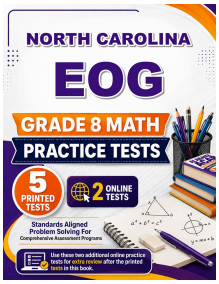
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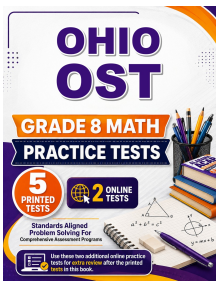
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