Estimating Expressions with Irrational Numbers

Reason with Radical Expressions

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Square roots do not distribute over sums: $\sqrt{9+16}=5$ but $\sqrt{9}+\sqrt{16}=7$. Likewise $3\sqrt{2}$ means 3 times $\sqrt{2}$, not $\sqrt{6}$. Test inequalities by exact squares when both sides are nonnegative. In reverse problems, isolate the root before squaring. For subtraction, $a<\sqrt{n}<b$ gives $d-b<d-\sqrt{n}<d-a$.

Worked example

Estimate $10-\sqrt{7}$ to the nearest tenth, checking endpoint order.

1. Square the trial bounds: $2.6457^2=6.99972849<7<7.00025764=2.6458^2$. Thus $2.6457<\sqrt{7}<2.6458$.

2. Substitute both bounds into $-1\sqrt{7}+10$. The endpoint values are $-1(2.6457)+(10)=7.3543$ and $-1(2.6458)+(10)=7.3542$.

3. Ordering these results gives $7.3542<-1\sqrt{7}+10<7.3543$. Both bounds round to 7.4, so the requested estimate is 7.4.

Answer: 7.4

Practice Problems

Show your thinking.

1. Correct $\sqrt{9}+\sqrt{16}=\sqrt{25}=5$ by evaluating the original terms.

2. A student uses $3\sqrt{5}=\sqrt{15}$. Estimate the original expression to the nearest tenth to correct the result.

3. A student substitutes $\sqrt{10}\approx 3.2$ and reports $6-\sqrt{10}\approx 2.80$. Find the correct nearest hundredth.

4. If $2.6<\sqrt{7}<2.7$, which bound is correct for $5-\sqrt{7}$: $(2.3, 2.4)$ or $(2.4, 2.5)$?

5. Find every positive integer n such that $9<2\sqrt{n}+1<11$.

6. Compare $2\sqrt{6}$ and 5 exactly, without decimal rounding.

7. If $1.7<\sqrt{3}<1.8$, bound $-2\sqrt{3}$. Explain the endpoint order.

8. A student rounds $\sqrt{8}$ to 3 before evaluating $2+\sqrt{8}$. Find the nearest-hundredth expression value without early rounding.

9. Find every whole number k such that $7<k+\sqrt{2}<8$. Use $1.4<\sqrt{2}<1.5$.

10. Use $3.16<\sqrt{10}<3.17$ to test the claim $2\sqrt{10}+1>7.4$.

Answer Key

1. 7, not 5.
2. 6.7
3. 2.84
4. $2.3 < 5 - \sqrt{7} < 2.4$.
5. $n = 17, 18, \ldots, 24$.
6. $2\sqrt{6} < 5$.
7. $-3.6 < -2\sqrt{3} < -3.4$.
8. 4.83
9. Only $k = 6$.
10. False; $7.32 < 2\sqrt{10} + 1 < 7.34$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Evaluate each principal root separately: $\sqrt{9} = 3$ and $\sqrt{16} = 4$, since $3^2 = 9$ and $4^2 = 16$.

2. The original sum is $3 + 4 = 7$. Combining radicands under addition gave 5, which is a different value; the square-root operation does not distribute over addition.

Problem 2

1. Square the trial bounds: $2.236^2 = 4.999696 < 5 < 5.00014321 = 2.2361^2$. Thus $2.236 < \sqrt{5} < 2.2361$.

2. Substitute both bounds into $3\sqrt{5} + 0$. The endpoint values are $3(2.236) + (0) = 6.708$ and $3(2.2361) + (0) = 6.7083$.

3. Ordering these results gives $6.708 < 3\sqrt{5} + 0 < 6.7083$. Both bounds round to 6.7, so the requested estimate is 6.7.

Problem 3

1. Square the trial bounds: $3.1622^2 = 9.99950884 < 10 < 10.00014129 = 3.1623^2$. Thus $3.1622 < \sqrt{10} < 3.1623$.

2. Substitute both bounds into $-1\sqrt{10} + 6$. The endpoint values are $-1(3.1622) + (6) = 2.8378$ and $-1(3.1623) + (6) = 2.8377$.

3. Ordering these results gives $2.8377 < -1\sqrt{10} + 6 < 2.8378$. Both bounds round to 2.84, so the requested estimate is 2.84.

Problem 4

1. Subtract the upper root bound for the lower expression bound: $5 - 2.7 = 2.3$. Subtraction reverses which root endpoint is used.

2. Subtract the lower root bound for the upper expression bound: $5 - 2.6 = 2.4$. Therefore the first interval, $(2.3, 2.4)$, is the correct choice.

Problem 5

1. Subtract 1 and divide by positive 2: $8 < 2\sqrt{n} < 10$ becomes $4 < \sqrt{n} < 5$.

2. Square the positive bounds to obtain $16 < n < 25$. The permitted integer radicands are 17 through 24; the two square endpoints are excluded.

Problem 6

1. Both quantities are positive, so their order is the same as the order of their squares. Compute $(2\sqrt{6})^2 = 4 \cdot 6 = 24$.

2. The other square is $5^2 = 25$. Since $24 < 25$, the original positive values satisfy $2\sqrt{6} < 5$; an approximate root is unnecessary.

Problem 7

1. Multiply both positive-root bounds by -2: $-2(1.7) = -3.4$ and $-2(1.8) = -3.6$. The negative multiplier reverses both inequality signs.

2. Put the smaller number first to obtain $-3.6 < -2\sqrt{3} < -3.4$. The more positive root endpoint produces the more negative expression endpoint.

Problem 8

1. Square the trial bounds: $2.8284^2 = 7.99984656 < 8 < 8.00041225 = 2.8285^2$. Thus $2.8284 < \sqrt{8} < 2.8285$.

2. Substitute both bounds into $1\sqrt{8} + 2$. The endpoint values are $1(2.8284) + (2) = 4.8284$ and $1(2.8285) + (2) = 4.8285$.

3. Ordering these results gives $4.8284 < 1\sqrt{8} + 2 < 4.8285$. Both bounds round to 4.83, so the requested estimate is 4.83.

Problem 9

1. The original inequality gives $7 - \sqrt{2} < k < 8 - \sqrt{2}$. The endpoints lie in $(5.5, 5.6)$ and $(6.5, 6.6)$ respectively.

2. Only the integer 6 lies between these endpoints. Verify directly: $7.4 < 6 + \sqrt{2} < 7.5$, which is wholly inside the required interval $(7, 8)$.

Problem 10

1. Multiply the given bounds by 2 and add 1: $2(3.16) + 1 = 7.32$ and $2(3.17) + 1 = 7.34$.

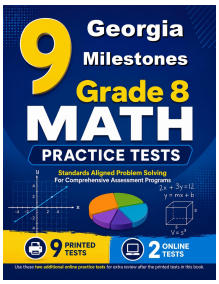
2. Thus $7.32 < 2\sqrt{10} + 1 < 7.34$. Since even the upper bound 7.34 is less than 7.4, the claimed inequality is false.

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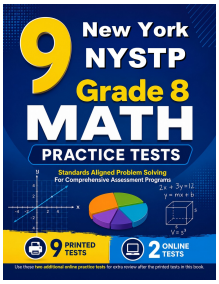
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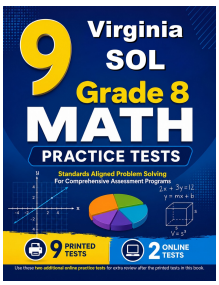
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