# Estimating Expressions with Irrational Numbers

Estimate the Whole Expression

**Name** _____ **Date** _____ **Score** _____ / 10

**QUICK REVIEW**

Estimate an entire expression before rounding. If $a < \sqrt{n} < b$ and $c > 0$, then $ca + d < c\sqrt{n} + d < cb + d$. A negative multiplier reverses endpoint order. For a difference of roots, the lower bound subtracts the larger second endpoint. Use enough root digits that both final bounds round identically; premature root rounding can change the answer.

**Worked example**

Estimate $2\sqrt{5} + 3$ to the nearest hundredth.

**1.** Square the trial bounds: $2.236^2 = 4.999696 < 5 < 5.00014321 = 2.2361^2$. Thus $2.236 < \sqrt{5} < 2.2361$.

**2.** Substitute both bounds into $2\sqrt{5} + 3$. The endpoint values are $2(2.236) + (3) = 7.472$ and $2(2.2361) + (3) = 7.4722$.

**3.** Ordering these results gives $7.472 < 2\sqrt{5} + 3 < 7.4722$. Both bounds round to $7.47$, so the requested estimate is $7.47$.

**Answer:** $7.47$

## Practice Problems

Show your thinking.

**1.** Estimate $2\sqrt{6} + 1$ to the nearest tenth.

**2.** Estimate $1\sqrt{11} - 2$ to the nearest hundredth.

**3.** Estimate $\frac{\sqrt{15}}{2}$ to the nearest hundredth.

**4.** Estimate $4\sqrt{3} - 1$ to the nearest tenth.

**5.** Estimate $8 - \sqrt{21}$ to the nearest hundredth. Keep the subtraction order.

**6.** Estimate $3\sqrt{35} + 2$ to the nearest integer.

**7.** Estimate $5 - 2\sqrt{2}$ to the nearest tenth.

**8.** Use $1.414 < \sqrt{2} < 1.415$ and $1.732 < \sqrt{3} < 1.733$ to bound $\sqrt{2} + \sqrt{3}$ and round to the nearest hundredth.

**9.** Use $2.236 < \sqrt{5} < 2.237$ and $1.414 < \sqrt{2} < 1.415$ to bound $\sqrt{5} - \sqrt{2}$. Round to the nearest hundredth.

**10.** Estimate $1 + \frac{\sqrt{7}}{4}$ to the nearest hundredth.

# Answer Key

1. $5.9$
2. $1.32$
3. $1.94$
4. $5.9$
5. $3.42$
6. $20$
7. $2.2$
8. $3.146 < \sqrt{2} + \sqrt{3} < 3.148$; $3.15$.
9. $0.821 < \sqrt{5} - \sqrt{2} < 0.823$; $0.82$.
10. $1.66$

## Step-by-Step Solutions

Follow each calculation, then check your own work.

### Problem 1

**1.** Square the trial bounds: $2.4494^2 = 5.99956036 < 6 < 6.00005025 = 2.4495^2$. Thus $2.4494 < \sqrt{6} < 2.4495$.

**2.** Substitute both bounds into $2\sqrt{6} + 1$. The endpoint values are $2(2.4494) + (1) = 5.8988$ and $2(2.4495) + (1) = 5.899$.

**3.** Ordering these results gives $5.8988 < 2\sqrt{6} + 1 < 5.899$. Both bounds round to $5.9$, so the requested estimate is $5.9$.

### Problem 2

**1.** Square the trial bounds: $3.3166^2 = 10.99983556 < 11 < 11.00049889 = 3.3167^2$. Thus $3.3166 < \sqrt{11} < 3.3167$.

**2.** Substitute both bounds into $1\sqrt{11} - 2$. The endpoint values are $1(3.3166) + (-2) = 1.3166$ and $1(3.3167) + (-2) = 1.3167$.

**3.** Ordering these results gives $1.3166 < 1\sqrt{11} - 2 < 1.3167$. Both bounds round to $1.32$, so the requested estimate is $1.32$.

### Problem 3

**1.** Square the trial bounds: $3.8729^2 = 14.99935441 < 15 < 15.000129 = 3.873^2$. Thus $3.8729 < \sqrt{15} < 3.873$.

**2.** Substitute both bounds into $\frac{1}{2}\sqrt{15} + 0$. The endpoint values are $\frac{1}{2}(3.8729) + (0) = 1.93645$ and $\frac{1}{2}(3.873) + (0) = 1.9365$.

**3.** Ordering these results gives $1.93645 < \frac{1}{2}\sqrt{15} + 0 < 1.9365$. Both bounds round to $1.94$, so the requested estimate is $1.94$.

### Problem 4

**1.** Square the trial bounds: $1.732^2 = 2.999824 < 3 < 3.00017041 = 1.7321^2$. Thus $1.732 < \sqrt{3} < 1.7321$.

**2.** Substitute both bounds into $4\sqrt{3} - 1$. The endpoint values are $4(1.732) + (-1) = 5.928$ and $4(1.7321) + (-1) = 5.9284$.

**3.** Ordering these results gives $5.928 < 4\sqrt{3} - 1 < 5.9284$. Both bounds round to $5.9$, so the requested estimate is $5.9$.

### Problem 5

**1.** Square the trial bounds: $4.5825^2 = 20.99930625 < 21 < 21.00022276 = 4.5826^2$. Thus $4.5825 < \sqrt{21} < 4.5826$.

**2.** Substitute both bounds into $-1\sqrt{21} + 8$. The endpoint values are $-1(4.5825) + (8) = 3.4175$ and $-1(4.5826) + (8) = 3.4174$.

**3.** Ordering these results gives $3.4174 < -1\sqrt{21} + 8 < 3.4175$. Both bounds round to $3.42$, so the requested estimate is $3.42$.

### Problem 6

**1.** Square the trial bounds: $5.916^2 = 34.999056 < 35 < 35.00023921 = 5.9161^2$. Thus $5.916 < \sqrt{35} < 5.9161$.

**2.** Substitute both bounds into $3\sqrt{35} + 2$. The endpoint values are $3(5.916) + (2) = 19.748$ and $3(5.9161) + (2) = 19.7483$.

**3.** Ordering these results gives $19.748 < 3\sqrt{35} + 2 < 19.7483$. Both bounds round to $20$, so the requested estimate is $20$.

### Problem 7

**1.** Square the trial bounds: $1.4142^2 = 1.99996164 < 2 < 2.00024449 = 1.4143^2$. Thus $1.4142 < \sqrt{2} < 1.4143$.

**2.** Substitute both bounds into $-2\sqrt{2} + 5$. The endpoint values are $-2(1.4142) + (5) = 2.1716$ and $-2(1.4143) + (5) = 2.1714$.

**3.** Ordering these results gives $2.1714 < -2\sqrt{2} + 5 < 2.1716$. Both bounds round to $2.2$, so the requested estimate is $2.2$.

### Problem 8

**1.** Add the supplied lower bounds: $1.414 + 1.732 = 3.146$. Adding inequalities is valid here, so the sum is greater than $3.146$.

**2.** Add the upper bounds: $1.415 + 1.733 = 3.148$. Thus $3.146 < \sqrt{2} + \sqrt{3} < 3.148$, and every value in this interval rounds to $3.15$.

### Problem 9

**1.** The smallest difference uses the lower first term and upper subtracted term: $2.236 - 1.415 = 0.821$. This gives the strict lower bound.

**2.** The largest uses the upper first term and lower subtracted term: $2.237 - 1.414 = 0.823$. The whole interval $(0.821, 0.823)$ rounds to $0.82$.

### Problem 10

**1.** Square the trial bounds: $2.6457^2 = 6.99972849 < 7 < 7.00025764 = 2.6458^2$. Thus $2.6457 < \sqrt{7} < 2.6458$.

**2.** Substitute both bounds into $\frac{1}{4}\sqrt{7} + 1$. The endpoint values are $\frac{1}{4}(2.6457) + (1) = 1.661425$ and $\frac{1}{4}(2.6458) + (1) = 1.66145$.

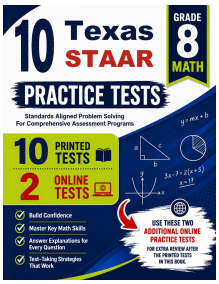
**3.** Ordering these results gives $1.661425 < \frac{1}{4}\sqrt{7} + 1 < 1.66145$. Both bounds round to $1.66$, so the requested estimate is $1.66$.

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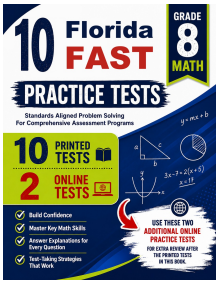

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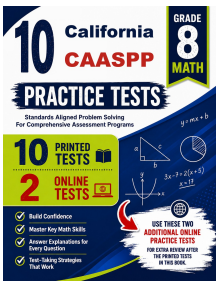

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