Counting Principle and Permutations

Apply the skill to decisions and designs

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

Choose a counting model from the design constraints. Distinct officer roles or ranked prizes use permutations; an unranked committee uses combinations. Multiply independent menu choices, but reduce later counts when objects cannot be reused. Fixed positions contribute one choice. A valid design count should describe complete outcomes exactly once, including any restrictions on the first symbol or on who fills a role.

Worked example

Example for form 3: assign two distinct roles to different people from a group of 7.

1. The positions are distinct, so order matters. Without reuse, the choice counts are $7, 6$ for the successive positions.

2. Multiply: $7 \cdot 6 = 42$. Swapping two selected people or objects changes their positions, so do not divide by the number of reorderings.

Answer: 42 arrangements.

Practice Problems

Show your thinking.

1. A club of 7 students elects a president, secretary, and treasurer, all different. Count officer assignments.

2. A club of 7 students sends an unranked delegation of 3. Count delegations.

3. Design a badge with 2 shapes, 4 colors, and 3 borders, one choice per feature. Count badge designs.

4. An access code has two digits from 0 through 9 followed by a letter from A through F. Digits may repeat and leading zero is allowed. Count codes.

5. A race has 8 entrants with no ties. Count possible first-second-third podiums.

6. A display has 5 distinct posters; one specified poster must be first. Count complete arrangements.

7. A librarian selects 2 of 6 new books for an unranked donation set. Count sets.

8. A lunch special has 3 mains and 2 sides. How many specials exist if each uses one main and one side?

9. A designer needs at least 30 distinct codes. Compare two nonrepeating positions from 6 symbols with two repeating positions from 5 symbols. Which meets the target?

10. A school needs 12 badge designs from one shape and one color per badge. It offers 3 shapes. What minimum number of colors is needed?

Answer Key

1. 210 arrangements.
2. 35 selections.
3. 24 outcomes.
4. 600 outcomes.
5. 336 arrangements.
6. 24 outcomes.
7. 15 selections.
8. 6 outcomes.
9. The 6-symbol nonrepeating design: 30 codes.
10. 4 colors.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. The positions are distinct, so order matters. Without reuse, the choice counts are $7, 6, 5$ for the successive positions.

2. Multiply: $7 \cdot 6 \cdot 5 = 210$. Swapping two selected people or objects changes their positions, so do not divide by the number of reorderings.

Problem 2

1. Counting ordered choices first gives $7 \cdot 6 \cdot 5 = 210$. But each selected group appears in $3! = 6$ different orders.

2. Order is irrelevant here, so divide out those repeats: $210/6 = 35$. This is a combination, not an assignment to distinct positions.

Problem 3

1. A design has one shape, one color, and one border. The successive choices are 2, 4, and 3, and every combination is permitted.

2. Multiply the number of choices at the successive stages: $2 \cdot 4 \cdot 3 = 24$. Each complete outcome uses one choice at every stage, so addition would count only separate choices, not completed outcomes.

Problem 4

1. The first digit has 10 choices, including zero. Repetition permits 10 second-digit choices, and the final letter has 6 choices.

2. Multiply the number of choices at the successive stages: $10 \cdot 10 \cdot 6 = 600$. Each complete outcome uses one choice at every stage, so addition would count only separate choices, not completed outcomes.

Problem 5

1. The positions are distinct, so order matters. Without reuse, the choice counts are $8, 7, 6$ for the successive positions.

2. Multiply: $8 \cdot 7 \cdot 6 = 336$. Swapping two selected people or objects changes their positions, so do not divide by the number of reorderings.

Problem 6

1. The required first poster has 1 choice. The other four distinct posters fill the remaining positions with $4, 3, 2, 1$ successive choices.

2. Multiply the number of choices at the successive stages: $1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Each complete outcome uses one choice at every stage, so addition would count only separate choices, not completed outcomes.

Problem 7

1. Counting ordered choices first gives $6 \cdot 5 = 30$. But each selected group appears in $2! = 2$ different orders.

2. Order is irrelevant here, so divide out those repeats: $30/2 = 15$. This is a combination, not an assignment to distinct positions.

Problem 8

1. Each of the 3 mains can be combined with either of the 2 sides. A complete special is a main-side pair, with no further restrictions.

2. Multiply the number of choices at the successive stages: $3 \cdot 2 = 6$. Each complete outcome uses one choice at every stage, so addition would count only separate choices, not completed outcomes.

Problem 9

1. The first design has $6(5) = 30$ ordered codes. The second permits repetition but has only $5(5) = 25$ ordered codes.

2. Since $30 \geq 30$ but $25 < 30$, only the six-symbol nonrepeating design meets the minimum. More permissive reuse does not compensate for every smaller symbol set.

Problem 10

1. With c colors, the design count is $3c$. The requirement $3c \geq 12$ gives $c \geq 12/3 = 4$.

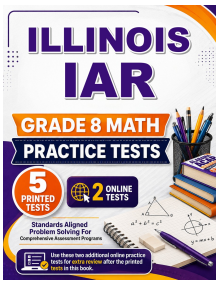
2. Four colors produce $3(4) = 12$ designs. Three colors would produce only $3(3) = 9$, so four is the minimum whole-number choice.

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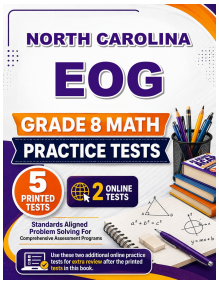
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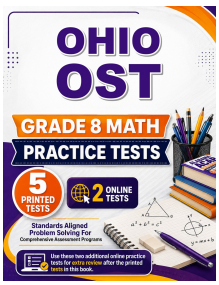
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