Theoretical Probability

Apply the skill to decisions and designs

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

For equally likely elementary outcomes, count favorable outcomes once and divide by all possible outcomes. A wheel with 5 equal sections, of which 2 are blue, has blue probability $2/5$. Complements total 1; count outcomes, not the size of their labels.

Worked example

Find probability of a multiple of 3 on a fair die.

1. Favorable labels are $3, 6$: two outcomes.
2. There are six equally likely labels.
3. $P = 2/6 = 1/3$; complement $4/6 = 2/3$.
Answer: $1/3$.

Practice Problems

Show your thinking.

1. A prize wheel has 12 equally likely sectors, with 4 winners and 8 losses. What is the fewest losing sectors that must become winners so winning is more likely than losing?

2. Wheel A has equally likely labels 1 through 10 and pays on multiples of 3. Wheel B has equally likely labels 1 through 12 and pays on odd labels below 7. Which wheel wins more often in theory, and by how much?

3. A bag has 10 equally likely numbered tickets, from 1 to 10. A win needs a number greater than 4 and odd. A report lists all numbers above 4 as wins. Correct the list and find the chance of no win.

4. A spinner designer wants exactly $1/2$ winning chance. Can nine equally likely sectors do it? If one sector is added, give a working number of winning and losing sectors.

5. A fair wheel has 5 red, 3 blue and 4 green sectors. One red sector is repainted blue. Compare the red and blue chances before and after. Does the chance of red or blue change?

6. Game A has 3 winning outcomes of 9 equally likely outcomes. Game B has 4 of 12. Are their initial chances equal? If each game changes one winning outcome to a loss, which has the higher chance?

7. A fair wheel has labels 1 through 12. It pays on prime labels. Is the chance at least $1/2$? Show the list of winners.

8. A fair 1–12 wheel currently pays for labels 4 through 8. A designer may extend the winning run by the same number of labels at each end. What is the smallest extension that makes the chance $3/4$?

9. A fair wheel uses labels 1 through 12. Blue wins when a label is not divisible by 3. Which is more likely: blue wins, or it lands on an even label? Count both, not just the labels that are in both groups.

10. A fair wheel uses labels 1 through 12. A player can choose to win on labels divisible by 2 or 3 (either condition), or to win only when both conditions hold. How much does allowing either condition increase the chance?

Answer Key

1. Repaint 3 losing sectors.
2. A by $1/20$.
3. Wins: $5, 7, 9$. No win: $7/10$.
4. No with 9; with 10, use 5 winners and 5 losses.
5. Red falls from $5/12$ to $4/12$; blue rises from $3/12$ to $4/12$. Red or blue stays $2/3$.
6. Initially both $1/3$. After the change B leads by $1/36$.
7. No; $5/12$.
8. Extend by 2 labels on each end, to 2 through 10.
9. Blue wins more often: $2/3$ versus $1/2$.
10. Increase $1/2$, from $1/6$ to $2/3$.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Half of 12 is 6; exactly 6 winning sectors would only tie the losing count.
2. The smallest winning count greater than 6 is 7.
3. Change $7 - 4 = 3$ losing sectors. The resulting $7/12$ exceeds $5/12$.

Problem 2

1. A pays on $3, 6, 9$: probability $3/10$.
2. B pays on $1, 3, 5$: probability $3/12 = 1/4$.
3. Compare using twentieths: $6/20 - 5/20 = 1/20$, favoring A.

Problem 3

1. The numbers above 4 are $5, 6, 7, 8, 9, 10$.
2. Keep only the odd labels: $5, 7, 9$, so 3 of 10 win.
3. The complement has $10 - 3 = 7$ tickets, giving $7/10$.

Problem 4

1. Exactly half of 9 is 4.5, not a whole number of sectors.
2. Adding one sector gives 10 sectors, so half is $10/2 = 5$.
3. Use 5 winning and 5 losing sectors: $5/10 = 1/2$.

Problem 5

1. The total stays 12. Red changes from 5 to 4 and blue from 3 to 4.
2. The respective chances therefore become $4/12$ and $4/12$.
3. Together the two colors have $5 + 3 = 8$ sectors before and $4 + 4 = 8$ after, so the combined chance stays $8/12 = 2/3$.

Problem 6

1. Initial chances simplify: $3/9 = 4/12 = 1/3$.
2. Change one winner in each without changing the totals: A is $2/9$ and B is $3/12 = 1/4$.
3. Compare with denominator 36: B is $9/36$, A is $8/36$. The gap is $1/36$.

Problem 7

1. The prime labels are $2, 3, 5, 7, 11$. The label 1 is not prime.
2. Five of the 12 equally likely labels win: $5/12$.
3. Half is $6/12$, so the chance is $1/12$ below the goal.

Problem 8

1. The current interval contains $8 - 4 + 1 = 5$ labels.
2. The goal needs $12(3/4) = 9$ winning labels, which is $9 - 5 = 4$ more.
3. Extend by $4/2 = 2$ on each end. Labels 2 through 10 give $9/12 = 3/4$.

Problem 9

1. The four multiples of 3 are $3, 6, 9, 12$, leaving $12 - 4 = 8$ blue outcomes.
2. There are 6 even labels: $2, 4, 6, 8, 10, 12$.
3. Compare $8/12$ with $6/12$; blue is higher by $2/12 = 1/6$.

Problem 10

1. Both conditions hold at $6, 12$: $2/12 = 1/6$.
2. Either condition holds at $2, 3, 4, 6, 8, 9, 10, 12$: $8/12 = 2/3$. Count 6 and 12 only once.
3. The chance increases by $(8 - 2)/12 = 6/12 = 1/2$.

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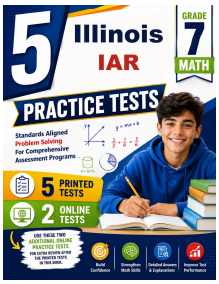

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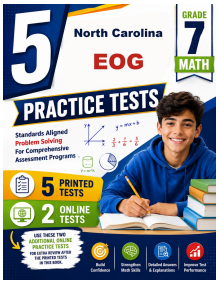

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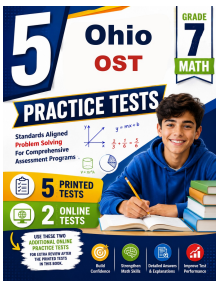

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