Theoretical Probability

Spot mistakes, reason, and work backward

Name _____ **Date** _____ **Score** _____ / 10

QUICK REVIEW

For equally likely elementary outcomes, count favorable outcomes once and divide by all possible outcomes. A wheel with 5 equal sections, of which 2 are blue, has blue probability $2/5$. Complements total 1; count outcomes, not the size of their labels.

Worked example

Find probability of a multiple of 3 on a fair die.

1. Favorable labels are $3, 6$: two outcomes.
2. There are six equally likely labels.
3. $P = 2/6 = 1/3$; complement $4/6 = 2/3$.
Answer: $1/3$.

Practice Problems

Show your thinking.

1. One ticket is drawn at random from the chosen bag. Which bag gives a better theoretical chance of a star, and by how much? Explain why star counts alone mislead.

Bag	Star	Circle
A	5	7
B	6	10

2. A spinner has 15 equal sectors, all amber, blue, or violet. Four sectors are blue, and $P(\text{not amber}) = 3/5$. Find the violet and amber counts.

3. The table counts tickets in a bag. Draw one at random. A star or striped ticket wins, including a striped star. A report counts 17 winners. What was counted twice? Find the actual win chance.

Border	Star	Circle	Triangle
Striped	3	5	2
Plain	4	1	5

4. Check four win models. A has 6 winners in 15 tickets; B gives 40%; C has 2 winners and 5 losers; D has 8 winners in 20 tickets. Which match A? Ticket draws are uniform.

5. A wheel has 8 equal sectors, 3 striped. In a test it lands striped 5 times in 10 spins. Give the theoretical and experimental rates. Which is higher and by how much?

6. The table labels the faces of a fair cube. A student assigns chance $1/3$ to each letter. Find $P(A)$ and $P(\text{not } A)$ and explain why the letters are not equally likely.

Face	1	2	3	4	5	6
Label	A	A	A	B	C	C

7. A bag has 8 red, 6 blue, and 10 green tickets. Draw one at random. Choose one winning color with a chance strictly between $1/4$ and $2/5$.

8. Bag A has 3 winning tickets out of 8; bag B has 5 out of 12. Draw uniformly. Can removing only losing tickets from A make its chance exactly equal to B?

9. Six tickets A through F are equally likely. Rule P accepts A, B, D; rule Q accepts B, D, E. A win needs exactly one rule. A student adds $3 + 3$ winners. Audit the list and find the win chance.

10. Can 30 equally likely tokens give a win chance of exactly $7/20$? If not, find the chances for the two nearest whole winning counts and compare their distances from the target.

Answer Key

1. Bag A by $1/24$.
2. 5 violet sectors and 6 amber sectors.
3. The 3 striped stars were counted twice. Win chance $7/10$.
4. B and D match A at $2/5$; C gives $2/7$.
5. Theoretical $3/8$; experimental $1/2$, higher by $1/8$.
6. $P(A) = 1/2$ and $P(\text{not } A) = 1/2$.
7. Red only: $1/3$.
8. No. A starts below B and jumps above it after one removal.
9. Only A and E win: $2/6 = 1/3$.
10. No. 10 winners give $1/3$ and 11 give $11/30$; both are $1/60$ from the target.

Step-by-Step Solutions

Follow each calculation, then check your own work.

Problem 1

1. Bag A gives $5/(5+7) = 5/12$. Bag B gives $6/(6+10) = 3/8$.
2. Compare $20/48$ with $18/48$. A leads by $20/48 - 18/48 = 2/48 = 1/24$.
3. B has more stars but also more total tickets. The fractions, not the raw counts, determine the chances.

Problem 2

1. Not amber includes blue and violet: $15(3/5) = 9$ sectors.
2. Violet has $9 - 4 = 5$ sectors; amber has $15 - 9 = 6$.
3. Check: $4 + 5 + 6 = 15$ and $(4+5)/15 = 3/5$.

Problem 3

1. There are $3 + 4 = 7$ stars and $3 + 5 + 2 = 10$ striped tickets. Both counts include the same 3 striped stars.
2. Count those once: $7 + 10 - 3 = 14$ distinct winners.
3. The rows contain 10 tickets each, so the chance is $14/20 = 7/10$.

Problem 4

1. A gives $6/15 = 2/5$.
2. B gives $40\% = 40/100 = 2/5$, and D gives $8/20 = 2/5$. Both match A.
3. C has $2 + 5 = 7$ total outcomes, so its chance is $2/7$, not $2/5$.

Problem 5

1. Use the sector model for theory: 3 of 8 equal sectors give $3/8$.
2. Use observed trials for the experimental rate: $5/10 = 1/2 = 4/8$.
3. The test rate is higher by $4/8 - 3/8 = 1/8$. A short test can differ from the model.

Problem 6

1. The 6 faces are equally likely, not the 3 distinct letters.
2. A appears on 3 faces, so $P(A) = 3/6 = 1/2$.
3. B appears once and C twice. The $1 + 2 = 3$ other faces give $P(\text{not } A) = 3/6 = 1/2$.

Problem 7

1. Total: $8 + 6 + 10 = 24$. Red gives $8/24 = 1/3$, blue $6/24 = 1/4$, and green $10/24 = 5/12$.
2. In sixtieths the limits are $15/60$ and $24/60$. Red is $20/60$, blue is $15/60$, and green is $25/60$.
3. Only red lies strictly inside: $15/60 < 20/60 < 24/60$.

Problem 8

1. Before removing any tickets, A gives $3/8 = 9/24$, below B at $5/12 = 10/24$.
2. After one removal, A gives $3/7 = 36/84$, above B at $35/84$.
3. Any further removal shrinks the total without changing the 3 winners, so A only increases. No whole number of removals works.

Problem 9

1. B and D are in both lists, so adding $3 + 3$ counts them twice. Neither can win under the exactly-one rule.
2. C and F meet neither rule. A meets only P, and E meets only Q.
3. The two winning tickets give probability $2/6 = 1/3$.

Problem 10

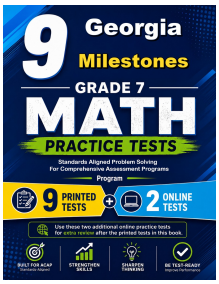
1. The exact target would need $30(7/20) = 10.5$ winners. Whole counts on either side are 10 and 11.
2. Their chances are $10/30 = 20/60$ and $11/30 = 22/60$, around the target $7/20 = 21/60$.
3. The gaps are $(21 - 20)/60 = 1/60$ and $(22 - 21)/60 = 1/60$.

TESTINAR

Keep Building Grade 7 Math Skills

Choose the practice book matched to your state assessment and turn worksheet practice into a complete test-prep plan.

All Grade 7 books



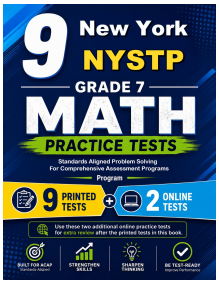
GEORGIA MILESTONES

9 Full-Length Math Practice Tests

Build Grade 7 accuracy, confidence, and test stamina with 9 printable practice tests.

Printable eBook • Instant PDF access

View eBook



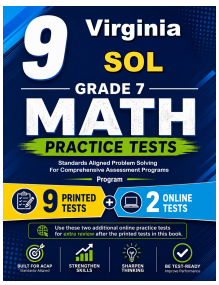
NEW YORK NYSTP

9 Full-Length Math Practice Tests

Build Grade 7 accuracy, confidence, and test stamina with 9 printable practice tests.

Printable eBook • Instant PDF access

View eBook



VIRGINIA SOL

9 Full-Length Math Practice Tests

Build Grade 7 accuracy, confidence, and test stamina with 9 printable practice tests.

Printable eBook • Instant PDF access

View eBook

Need a different state?
Testinar has Grade 7 math practice books for state assessments across the country.

TESTINAR.COM/GRADES/7