The Ultimate Step by Step Guide to Acing AP Calculus AB

Test taker's #1 Choice

AP Calculus AB

2026

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- Fun, Interactive, and Concrete Learning

for Beginners

Reza Nazari

✓Recommended by teachers & students

AP Calculus AB for Beginners

The Ultimate Step by Step Guide to Preparing for the AP Calculus AB Exam

By

Reza Nazari

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One Sided Limits

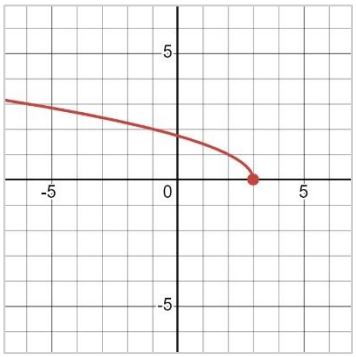
- One-sided limits are used to analyze the behavior of a function as it approaches a specific point from either the left or the right side.
- This can be helpful when dealing with functions that have discontinuities, sharp turns, or asymptotic behavior.
- One-sided limits are particularly useful in determining the continuity of a function.
- If the $f(x)$ function gets closer and closer to L when we approach a with x values greater but sufficiently close to it, we call L the one-sided limit of $f(x)$ and we show it as: $\lim_{x\to a^+} f(x) = L$.

  Similarly, if we approach a from the left, we show the one-sided limit as: $\lim_{x\to a^-} f(x) = L$.
- To discuss whether a function has a right limit at point a, it must be defined at the right neighborhood of a (same goes for a left neighborhood)
- Note that for the limit of a function to exist at a point, both left and right limits must exist and be equal. But it is a common convention in mathematics that if a function has a limit only from one side (either the left side or the right side) at a particular point, then the limit of the function at that point is equal to the limit from that specific side. Denoted as: $\lim_{x\to a} f(x) = \lim_{x\to a^+} f(x)$ or $\lim_{x\to a} f(x) = \lim_{x\to a^-} f(x)$.

  This convention holds as long as the function has a limit from at least one side and the other side does not produce a contradictory or different limit.

Example:

Solve: $\lim_{x\to 3} \sqrt{3-x}$.



Solution: This function is undefined for $x > 3$, so if we were to find the limit, we couldn't approach 3 from the right side. So, we can only get closer from $x < 3$. So, the limit for our function at $x = 3$ does not exist. But based on the convention mentioned earlier, we say the limit exists and is equal to: $\lim_{x\to 3^-} \sqrt{3-x} = 0$.

bit.ly/3GCSh5D
Find more at

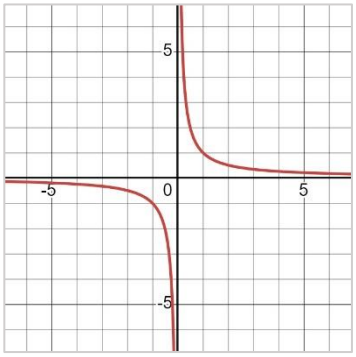
Limit at Infinity

- The limit at infinity describes how a function behaves as the variable approaches positive or negative infinity, meaning it examines how the function approaches a specific value or diverges as x becomes arbitrarily large (both towards negative and positive infinity).
- To evaluate the limit, we analyze the behavior of the function for large values of x.
- It is mathematically denoted as $\lim_{x\to\infty} f(x) = L$, where L is a real Number.
- If x approaches negative infinity, we write it as: $\lim_{x\to-\infty} f(x) = L$.
- If the function grows without bound or oscillates as x approaches infinity, the limit is said to be infinite and undefined, respectively.

Examples:

Example 1. Solve: $\lim_{x\to\infty} \frac{1}{x}$.

Solution: As x becomes larger and larger, the value of $\frac{1}{x}$ approaches 0. Therefore, the limit of $\frac{1}{x}$ as x approaches infinity is 0.

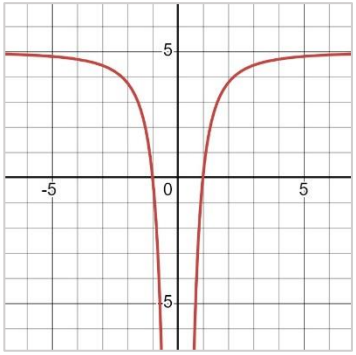


You can see this from the graph of this function:

As we increase the x values, the y or $f(x)$ get closer and closer to 0, no matter if we increase the positive x (getting more positive) or if we decrease the negative x (getting more negative).

Example 2. Solve: $\lim_{x\to\infty} -\frac{5}{x^2} + 5$.

Solution: Similar to the previous example, but this time, the graph looks like this:

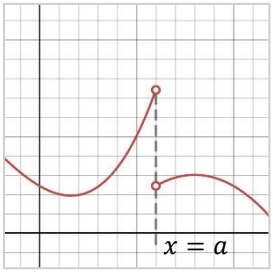


So, as x values increase (or decrease), y values get closer and closer to 5 (Because the $-\frac{5}{x^2}$ part, gets closer to zero as x grows). That's the limit of this function at infinity.

Find more at bit.ly/478n7h

Continuity at a Point

- We say the function $f(x)$ is continuous at point a , if the limit at a, exists and is equal to the result of $f(x)$ at a: $\lim_{x\to a} f(x) = f(a)$
- If the left-hand limit differs from the right-hand limit, it indicates a discontinuity or some form of jump in the function.
- For a function to be continuous at a point, you should be able to draw its graph without lifting your pen.
- If a function is continuous at all points in its domain, we call it a continuous function. Constant, Linear, Polynomial, Exponential, and Trigonometric functions are among the continuous functions.
- There are different types of discontinuity. Here are the common ones:
  - **Point Discontinuity (Removable Discontinuity):** Limit exists but function undefined or differs from limit at that point.
  - **Jump Discontinuity:** Left-hand and right-hand limits exist but are not equal.
  - **Infinite Discontinuity:** Function approaches infinity near the point.
  - **Essential (oscillating) Discontinuity:** Function oscillates, not settling to a single value or infinity.
- Here we have an example of a discontinuous function (jump). As we approach a from left and right, we get two different results. So $f(x)$ can't be equal to $f(a)$.



Examples:

Example 1. $\lim_{x\to 2} 7x - 5$

Solution: Assuming the function is continuous, it should have a limit equal to $f(2) = 7 \times 2 - 5 = 9$

1.9	1.99	1.999	2	2.001	2.01	2.1
8.3	8.93	8.993	9	9.007	9.07	9.7

Limit indeed approaches 9, so the function is continuous at $x = 2$

Example 2. Show that this function is not continuous for any a values:

$$f(x) = \begin{cases} (2ax+1)^2 \ for\ x > 1 \\ x^3 - 2x^2 \quad for\ x \le 1 \end{cases}$$

Solution: Assume each piece of this function is continuous, let's see what happens at $x = 1$ itself:

For 1^+: $(2ax+1)^2 = (2a+1)^2$, For 1^-: $x^3 - 2x^2 = 1^3 - 2(1)^2 = 1 - 2 = -1$

For $f(x)$ to be continuous, $(2a+1)^2$ must be equal to -1 which can't be, because the square of a number is always positive.

bit.ly/3RbGMXQ
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